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Top Quark Condensates and the Symmetry Breaking of the Electroweak Interactions [1]

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1. Introduction

A close physical analogy exists between the phenomena of electroweak symmetry breaking and superconductivity. In superconductivity the photon acquires a mass in a gauge invariant way, while in electroweak symmetry breaking the W and Z bosons acquire masses. Ginzburg and Landau [1] gave a phenomenological description of superconductivity by introducing a complex scalar field with a potential that caused the field to develop a Bose condensate, or in the nomenclature of particle physics, a "vacuum expectation value" (VEV). Particle physicists recognize this as spontaneous symmetry breaking in the abelian Higgs model. This is similar to the mode of description of electroweak symmetry breaking employed in the standard model in which a weak isospin–$\frac{1}{2}$ Higgs field develops a vacuum expectation value. Fermions acquire their masses from postulated couplings to the Higgs field and its VEV. Ultimately in the case of the superconductor a realistic dynamical mechanism was discovered in which the condensate of the scalar field is replaced by a condensate of dynamically paired electrons, through phonon interactions, in the BCS theory [2].

Many physicists believe that the true symmetry breaking of the electroweak interactions involves a dynamical mechanism in analogy to the BCS theory. The most celebrated mechanism is that of technicolor [3] and involves a new strong interaction that pairs techniquarks to form a weak isospin–$\frac{1}{2}$ condensate. This is, in the earliest version, a well understood dynamical mechanism for electroweak symmetry breaking, by analogy to the well–known chiral symmetry breaking in QCD, but it does not account for the masses of quarks and leptons. To give masses to the elementary fermions one must extend technicolor to a larger, broken symmetry group, known as "extended technicolor" [4], and here one encounters difficulties. The key problem is

[1] To appear in "Heavy Flavors," ed. A. Buras and M. Lindner, *World Scientific Pub. Co.*

that quarks cannot be too heavy in extended technicolor schemes without simultaneously generating large, unwanted $\Delta S = 2$ interactions, thus too large a $K_L - K_S$ mass difference. A naive estimate of the upper limit on a quark mass in standard extended technicolor is $m_{quark} \lesssim 100$ MeV. In a more recent version, "walking extended technicolor" [5], one might obtain larger masses, $m_{quark} \lesssim 10$ GeV. The top quark, however, is now known to be more massive than 90 GeV [6]. This is a severe problem for the technicolor scenarios which naturally favor a large electroweak breaking scale and comparatively small fermion masses.

In the standard model the current CDF lower limit implies that the top quark coupling to the elementary Higgs scalar is large, at least 20 times larger than that of the b quark. The coupling constant, $g_t = m_t/(175\ GeV)$, is now bounded below by $g_t \gtrsim 0.5$. The fact that the top quark is an elementary fermion with a mass scale of order the electroweak symmetry breaking scale suggests a dramatic new possibility: *the symmetry breakdown of the standard model may be a dynamical mechanism which intimately involves the top quark.* To implement this idea we will postulate new dynamics in which the top quark forms a condensate, for example $\langle \bar{t}_L t_R + h.c. \rangle$, which has the correct electroweak isospin$-\frac{1}{2}$ quantum number. Thus, in this scheme the top quark itself plays the role of a techniquark.

There is clearly much uncertainty in the specific new dynamics leading to top quark condensation. As a first step toward a full theory one can implement directly an effective Nambu–Jona-Lasinio (NJL) [7] mechanism in which an effective $SU(3) \times SU(2) \times U(1)$ invariant four–fermion interaction associated with a high energy scale, $\Lambda \sim G^{-1/2}$, is postulated [8, 9]:

$$\mathcal{L} = \mathcal{L}_{kinetic} + G(\overline{\Psi}_L^{ia} t_{Ra})(\bar{t}_R^{b} \Psi_{Lib}) \tag{1}$$

where i runs over $SU(2)_L$ indices, (a, b) run over color indices, and $\mathcal{L}_{kinetic}$ contains the usual gauge invariant fermion and gauge boson kinetic terms. There is no elementary Higgs field in $\mathcal{L}$. If $G > 0$ the interaction is attractive, and for sufficiently large G the four–fermion interaction triggers the formation of a low energy condensate, $\langle \bar{t}t \rangle$, which breaks $SU(2) \times U(1) \to U(1)$. The bootstrapping of the symmetry breaking mechanism to the top quark produces the requisite Nambu–Goldstone bosons associated with spontaneous symmetry breaking (which ultimately become the longitudinal components of W and Z), and also a composite particle which behaves identically to a fundamental Higgs boson at low energies.

By virtue of its economy this theory leads to new predictions which are testable in the near future. In particular, we are able to derive renormalization group improved predictions for m_{top} and m_{Higgs} (the composite $\bar{t}t$ Higgs boson) in this scheme, and we find, not surprisingly, that m_{top} is of order the weak scale. The results are very weakly dependent upon Λ; for example, with $\Lambda \sim 10^{15}$ GeV we find in the minimal scheme $m_{top} \approx 230$ GeV and $m_{Higgs} \sim 260$ GeV [9]. Yet another result, albeit not experimentaly accessible in the foreseeable future, is that the nonminimal coupling of the composite Higgs boson to gravity is determined, and we find the conformal value,

$\xi = 1/6$ as a general consequence of compositeness in the NJL model [10].

Thus, this model differs from technicolor at the outset in implying that at least one fermion, that associated with the electroweak condensate, must be heavy while the others are light. The usual Cabibbo–Kobayashi–Maskawa mixing angle structure and light fermion mass spectrum are readily accomodated, but predictions of mixing angles and the light quark masses are not derivable until one specifies the dynamics at the scale Λ more precisely. The usual one–Higgs–doublet standard model emerges as the low energy effective Lagrangian, but with new constraints that lead to the nontrivial predictions for m_{top}, m_{Higgs} and ξ.

The NJL [7] model is conventionally treated in a large N_{color} approximation, keeping only the effects of fermion loops. We will summarize that treatment in the next section. However, one can equivalently analyze the model using the renormalization group (RG) exclusively. This involves studying the effective Lagrangian and the evolution of its parameters as we vary the scale of physics, μ. At the high energies, $\mu \sim G^{-1/2}$ our theory is described by the four–fermion interaction of eq.(1). At low energies it contains a dynamical, composite weak isodoublet Higgs boson with self interactions and a Higgs–Yukawa coupling to the top quark. We must then understand how to "match" the low energy Lagrangian onto the high energy Lagrangian. The conditions that define this matching are called the "compositeness conditions" [9, 11]. The compositeness conditions are equivalent to boundary conditions near the scale Λ on the renormalization group equations. With the correct compositeness conditions we easily recover the conventional NJL results in the large–N limit.

The compositeness conditions are actually more powerful; they may be applied to *the full theory*, which goes beyond the large–N approximation and includes the effects of gauge boson and internal Higgs boson lines, *etc.* Certain special renormalization group trajectories, *i.e.*, those satisfying the compositeness conditions, are thereby associated with the existence of composite structure. These lead to the precise RG improved predictions for m_{top}, m_{Higgs}, and ξ, which are very insensitive to the scale of new physics, $\Lambda \sim G^{-1/2}$ [9].

The composite theory is effectively a strongly coupled (Higgs–Yukawa and quartic Higgs couplings) standard model near the scale Λ. The low energy predictions that emerge are found to be governed in each case by infrared renormalization group fixed points [12, 13]. In particular, compositeness is associated with the infrared fixed points as formulated in ref.[13]. These fixed points are universal low energy values of the coupling constants that arise from arbitrarily large values at high energies. Because the low energy values are insensitive to a wide range of initial values, the compositeness predictions are robust, and largely insensitive to the precise details of the high energy theory. For example, the top quark is predicted to lie near 230 GeV with the Higgs near 260 GeV, and $\xi = 1/6$ for a composite scale within several orders of magnitude of, $\Lambda \sim 10^{15}$ GeV.

How robust are the compositeness conditions and hence the predictions of a theory as in eq.(1)? One can follow Suzuki [14], [15] and consider the sensitivity of the results to the presence of generic higher dimension operators. Again, owing to the infrared fixed points, the results are found to be very insensitive to higher dimension operators (in ref.[15] arbitrarily large coefficients of these operators are allowed and it is claimed that the infrared predictions of the composite theory can be modified; we will return to this issue in section 4.2). It is important to realize, however, that the theory of eq.(1) cannot be viewed as fundamental. One is therefore challenged to construct models in which eq.(1) emerges as the effective theory at the scale Λ. With a wide class of such models we can compute the strength of irrelevant operators.

We will give a discussion of a "topcolor" model in analogy to minimal technicolor [16]. While minimal technicolor is a theory which naturally breaks $SU(2) \times U(1)$ but leaves the fermions massless, minimal topcolor breaks the electroweak interactions with a dynamical top condensate, while leaving all other quarks and leptons massless. This may be a better point of departure for the construction of extended models in which all quarks and leptons receive masses, however this is a new subject and we will not pursue the development of detailed schemes in this paper.

In the end we face the fundamental problem of "naturalness," *i.e.*, how to evade significant fine–tuning of the theory. We will summarize two avenues: (1) SUSY generalizations of the minimal model [17, 18] in which supersymmetry protects the gap equation from having to fine–tune large quadratic divergences; and (2) Fourth generation schemes [19] in which the scale Λ is simply taken near ~ 1 TeV. There are, of course, other logical possibilities, *e.g.*, perhaps in some models various additional states contribute so that $\Lambda \sim 1$ TeV becomes acceptable (see *e.g.*, [20]).

2. Analysis in Fermion Bubble Approximation

The present discussion summarizes how the dynamical symmetry breaking mechanism through top quark condensation operates in the approximation of keeping only fermionic loops, or, equivalently, to leading order in $1/N_c$ with the QCD coupling constant set to zero. The "bare" relationships emerge between the composite Higgs boson, top quark and W boson masses. These relationships are only approximate, and in Section 4 we will give the precise predictions, after abstracting the compositeness conditions to the full theory.

2.1 Gap Equation

We will begin by summing the planar bubble diagrams in which the four–fermion interaction of eq.(1) is iterated. We first consider the solution to the gap equation for the induced top quark mass. The gap equation is a self–consistent Schwinger–Dyson equation for an induced fermion mass term in the *ground state* of the field theory. It is often useful to think in terms of a variational calculation of the Hamiltonian expectation value in a trial groundstate wave–functional in which the elementary fermionic excitations have a mass $m(q^2)$.

If a nonzero value of $m(q^2)$ can be found, then the Dirac sea is "pushed down in energy," and the nontrivial solution is the preferred solution of lower energy. In the case of the NJL model we can easily obtain the induced mass gap m_t in the presence of a cut–off Λ in loop momentum. The gap equation is indicated in Fig.(1) and has the form:

$$\begin{aligned} m_t &= -\frac{1}{2} G \langle \bar{t} t \rangle \\ &= 2GN_c m_t \frac{i}{(2\pi)^4} \int_0^\Lambda d^4 l \, (l^2 - m_t^2)^{-1} \end{aligned} \tag{2}$$

The result of evaluating eq.(2) with the momentum space cut–off Λ is:

$$G^{-1} = \frac{N_c}{8\pi^2} \left(\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2) \right). \tag{3}$$

which has solutions for sufficiently strong coupling, $G \geq G_c = 8\pi^2/N_c\Lambda^2$ where G_c is the "critical" coupling constant.

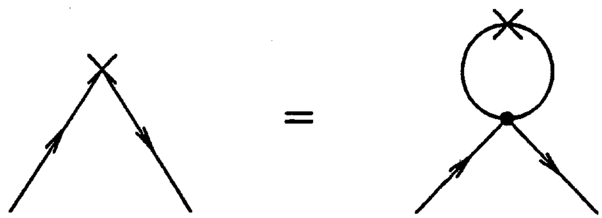

Figure (1): The gap equation.

Essentially Λ is the scale at which the four–fermion interaction softens into some kind of exchange potential (as in the "topcolor" model discussed in Section 5.) and $G = g^2/\Lambda^2$ involves a strong dimensionless coupling constant g. Hence we should regard G and Λ as fundamental parameters of the theory and we solve eq.(3) for m_t. Normally, for very large Λ, perhaps of order the GUT scale 10^{15} GeV, we would expect the solution of this equation to produce a large mass, $m_t \sim \Lambda$ in the broken symmetry phase. We see that a solution for $m_t \sim M_W$, for such large Λ, constitutes a fine–tuning problem in that $G^{-1} - G_c^{-1}$ must then be very small. This we will see below is the usual fine–tuning or gauge hierarchy problem of the standard model. The gap equation contains a quadratic divergence, corresponding to the usual Higgs mass quadratic divergence in the standard model. However, the fine–tuning problem will be isolated in the gap equation, *i.e.*, once we tune G to admit the desirable solution we need cancel no other quadratic divergences in other amplitudes.

2.2 Scalar and Nambu–Goldstone Modes

Let us now assume that the parameters, G, Λ admit a solution for m_t to the gap equation, eq.(2). We then consider the T–matrix element for the s–wave scattering of t and $\bar{t}$ in large–N, or equivalently, the sum of scalar channel fermion bubbles of Fig.(2) as generated by the interaction eq.(1):

$$\Gamma_s(p^2) = -\frac{1}{2}G - (\frac{1}{2}G)^2 i \int d^4x \; e^{ipx} \, \langle T \; \bar{t}t(0) \; \bar{t}t(x)\rangle_{connected} + \cdots \tag{4}$$

We may sum this series to obtain:

$$\begin{aligned}\Gamma_s(p^2) \;\; &= \;\; -\frac{1}{2}G\left[1 - 2GN_c\frac{i}{(2\pi)^4}\int d^4l\,(l^2 - m_t^2)^{-1}\right.\\ &\left. -GN_c(4m_t^2 - p^2)\frac{i}{(2\pi)^4}\int d^4l\,(l^2 - m_t^2)^{-1}((p+l)^2 - m_t^2)^{-1}\right]^{-1}\end{aligned} \tag{5}$$

Here the second and third terms in the denominator of eq.(5) come from a rearrangement of the terms in the numerator of the Feynman loop–integral and a shift of the the loop momentum for the fermions. We thus see that the first two terms in the denominator cancel by virtue of the gap equation eq.(2). Thus, performing the loop integrals we arrive at a result:

$$\Gamma_s(p^2) = -\frac{1}{2N_c}\left[(4m_t^2 - p^2)(4\pi)^{-2}\int_0^1 dx \log\left\{\Lambda^2/(m_t^2 - x(1-x)p^2)\right\}\right]^{-1} \tag{6}$$

Γ_s is the propagator for the dynamically generated boundstate, a scalar composite of $\bar{t}t$. In particular, owing to the pole at $p^2 = 4m_t^2$, we see that the theory predicts a scalar boundstate with a mass of $2m_t$ [7, 8, 9]. This is a standard result in the Nambu–Jona-Lasinio model. We emphasize that this boundstate is the physically observable low energy Higgs boson. At this stage the prediction holds only to leading order in $1/N_c$ in the absence of gauge boson corrections.

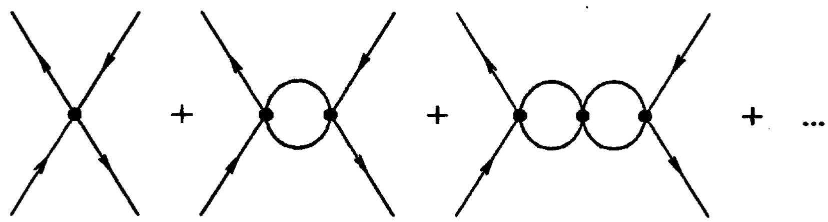

Figure (2): Sum of scalar channel fermion bubbles.

This physical particle is a boundstate of $\bar{t}t$, arising by the attractive four–fermion interaction at the scale Λ of eq.(1). One might be inclined to think that this is a loosely bound state, since it lies on the threshold for open $\bar{t}t$ and apparently has vanishing "binding energy" to this order. However, this is *not a nonrelativistic boundstate*, and normal nonrelativistic potential model intuition does not apply. The 0^+ state is present in the spectrum on all scales between $2m_t$ and Λ, owing to the $\log(\Lambda^2/p^2)$ behavior of eq.(6), and ceases to be a boundstate on scales $\gtrsim \Lambda$.

The prediction $m_H = 2m_t$ cannot be viewed at this stage as a very precise one, arising only in leading order in $1/N_c$ and neglecting all other interactions. We will give a more precise determination of its mass in Section 4, upon considering the full renormalization group behavior of the complete theory.

Since this mechanism represents a dynamical breaking of the continuous $SU(2) \times U(1)$ symmetry, it must imply the existence of Nambu–Goldstone modes. Moreover, the symmetry breaking transforms as $I = \frac{1}{2}$ and will produce the same spectrum of Nambu–Goldstone bosons as in the standard model Higgs–sector. A massless pole thus appears in the bubble sum for the neutral pseudo-scalar channel:

$$\Gamma_p(p^2) = -\frac{1}{2}G - (\frac{1}{2}G)^2 i \int d^4x \; e^{ipx} \, \langle T \; \bar{t}\gamma_5 t(0) \; \bar{t}\gamma_5 t(x) \rangle_{connected} + \ldots \tag{7}$$

By similar manipulations as in eq.(5) and use of the gap equation we find the result:

$$\Gamma_p(p^2) = \frac{1}{2N_c} \left[(p^2)(4\pi)^{-2} \int_0^1 dx \; \log \left\{ \Lambda^2/(m_t^2 - x(1-x)p^2) \right\} \right]^{-1} \tag{8}$$

and the Nambu–Goldstone pole at $p^2 = 0$ is seen to occur explicitly.

Moreover, charged Nambu–Goldstone modes appear in the flavored channels corresponding to the quantum numbers of the W boson:

$$\Gamma_F = -\frac{1}{4}G - (\frac{1}{4}G)^2 i \int d^4x\ e^{ipx} \left\langle T\ \bar{b}(1+\gamma_5)t(0)\ \bar{t}(1-\gamma_5)b(x)\right\rangle_{connected} + \dots \tag{9}$$

whence:

$$\Gamma_F(p^2) = \frac{1}{8N_c}\left[(p^2)(4\pi)^{-2}\int_0^1 dx(1-x)\log\left\{\Lambda^2/((1-x)m_t^2 - x(1-x)p^2)\right\}\right]^{-1} \tag{10}$$

where we have assumed $m_b \approx 0$.

2.3 Vector Bosons

Thus far we have considered only a conventional Nambu–Jona-Lasinio model for the symmetry group $SU(2)\times U(1)$ in the absence of gauge fields. Now let us consider the model with the gauge coupling constants restored. Of course, we have a dynamical Higgs–mechanism and the gauge bosons acquire masses and longitudinal degrees of freedom by "eating" the dynamically generated Nambu–Goldstone bosons. We obtain a second prediction of the theory in the form of a relation between the W boson mass and the top quark mass.

Consider the inverse propagator of a gauge bosons. We rescale fields to bring the gauge coupling constants into the gauge boson kinetic terms, *i.e.*, we write the kinetic terms in the form $(-1/4g^2)(F_{\mu\nu})^2$. The gauge boson fields should be viewed as background classical fields and we thus need specify no gauge fixing at this stage. Thus, for the W boson we have:

$$\frac{1}{g_2^2}D^W_{\mu\nu}(p)^{-1} = \frac{1}{g_2^2}(p_\mu p_\nu - g_{\mu\nu}p^2) + \frac{i}{2}\int d^4x\ \left\langle T\ \bar{t}_L\gamma_\mu b_L(0)\ \bar{b}_L\gamma_\nu t_L(x)\right\rangle \tag{11}$$

where g_2 is the $SU(2)$ coupling constant. For the T–ordered product we again expand in the interaction Lagrangian of eq.(1) and sum the planar bubbles, Fig.(3). We assume the top quark has a mass satisfying eq.(2), and the gap equation is satisfied in the loop expansion, which maintains the gauge invariance. This sum can thus be written in terms of the flavor bubbles evaluated in eq.(10).

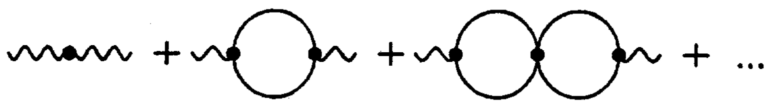

Figure (3): Vector boson inverse propagator.

It is useful to write the induced inverse W boson propagator in the form:

$$\frac{1}{g_2^2}D^W_{\mu\nu}(p)^{-1} = (p_\mu p_\nu/p^2 - g_{\mu\nu})\left[\frac{1}{\overline{g}_2^2(p^2)}p^2 - \overline{f}^2(p^2)\right]. \tag{12}$$

Note the transverse structure of the inverse propagator reflecting the gauge invariance of the theory. In effect, by integrating over the top and bottom quarks we have integrated out the longitudinal mode of the W boson, which brings the inverse propagator into the transverse form. The W boson mass is the solution to the mass–shell condition:

$$M_W^2 = p^2 = \overline{g}_2^2(p^2)\overline{f}^2(p^2) \tag{13}$$

while the Fermi constant is the zero–momentum expression:

$$\frac{G_F}{\sqrt{2}} = \frac{1}{8\overline{f}^2(0)} \tag{14}$$

Note, therefore, that our normalization conventions relate $\overline{f}(0)$ to the standard model Higgs vacuum expectation value v as follows: $v = (G_F\sqrt{2})^{-1/2} = 246\ GeV\ = 2\overline{f}(0)$. In the bubble approximation we find for one generation of quarks, t and b:

$$\begin{aligned}\frac{1}{\overline{g}_2^2(p^2)} &= \frac{1}{g_2^2} + N_c(4\pi)^{-2}\int_0^1 dx\ 2x(1-x) \\ &\qquad \times \log\left\{\Lambda^2/(xm_b^2 + (1-x)m_t^2 - x(1-x)p^2)\right\}\end{aligned} \tag{15}$$

and:

$$\begin{aligned}\overline{f}^2(p^2) &= N_c(4\pi)^{-2}\int_0^1 dx\ (xm_b^2 + (1-x)m_t^2) \\ &\qquad \times \log\left\{\Lambda^2/(xm_b^2 + (1-x)m_t^2 - x(1-x)p^2)\right\}\end{aligned} \tag{16}$$

Analogous results are obtained for the neutral gauge boson masses. We may consider the inverse propagator of the neutral gauge bosons as a 2×2 matrix of the form:

$$
\begin{aligned}
\frac{1}{g_i g_j} D^0_{\mu\nu}(p)^{-1} &= \begin{bmatrix} 1/g_2^2 & 0 \\ 0 & 1/g_1^2 \end{bmatrix} (p^\mu p^\nu - g^{\mu\nu} p^2) \\
&+ \frac{1}{2} i \int d^4x \begin{bmatrix} \left\langle T\, j^3_\mu(0)\, j^3_\nu(x) \right\rangle & \left\langle T\, j^3_\mu(0)\, j^0_\nu(x) \right\rangle \\ \left\langle T\, j^0_\mu(0)\, j^3_\nu(x) \right\rangle & \left\langle T\, j^0_\mu(0)\, j^0_\nu(x) \right\rangle \end{bmatrix} \qquad (17)
\end{aligned}
$$

where g_1 is the $U(1)$ coupling constant. Here the currents are the usual $SU(2)$ and $U(1)$ neutral currents in the unmixed basis:

$$
j^3_\mu = \bar{t}_L \gamma_\mu t_L - \bar{b}_L \gamma_\mu b_L \qquad (18)
$$

$$
j^0_\mu = \frac{1}{3}(\bar{t}_L \gamma_\mu t_L + \bar{b}_L \gamma_\mu b_L) + \frac{4}{3}(\bar{t}_R \gamma_\mu t_R) - \frac{2}{3}(\bar{b}_R \gamma_\mu b_R) \qquad (19)
$$

and the numerical factors in the individual terms of j^0_μ are the $U(1)$ weak–hypercharges. Again we expand in the interaction Lagrangian of eq.(1.1) and sum the planar bubbles, Fig.(3). This can be evaluated to yield:

$$
\frac{1}{g_i g_j} D^0_{\mu\nu}(p)^{-1} = (p^\mu p^\nu / p^2 - g^{\mu\nu}) \left\{ \begin{bmatrix} 1/g_2^2(p^2) & 0 \\ 0 & 1/g_1^2(p^2) \end{bmatrix} p^2 - \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} f^2(p^2) \right\} \qquad (20)
$$

where:

$$
\begin{aligned}
\frac{1}{g_2^2(p^2)} &= \frac{1}{g_2^2} + \frac{1}{2}(4\pi)^{-2} \int_0^1 dx\; 2x(1-x) \left\{ \frac{4}{3} N_c \log \left\{ \Lambda^2/(m_t^2 - x(1-x)p^2) \right\} \right. \\
&\left. + \frac{2}{3} N_c \log \left\{ \Lambda^2/(m_b^2 - x(1-x)p^2) \right\} \right\} \qquad (21)
\end{aligned}
$$

and:

$$
\begin{aligned}
\frac{1}{g_1^2(p^2)} &= \frac{1}{g_1^2} + \frac{1}{2}(4\pi)^{-2} \int_0^1 dx\; 2x(1-x) \left\{ \frac{20}{9} N_c \log \left\{ \Lambda^2/(m_t^2 - x(1-x)p^2) \right\} \right. \\
&\left. + \frac{2}{9} N_c \log \left\{ \Lambda^2/(m_b^2 - x(1-x)p^2) \right\} \right\} \qquad (22)
\end{aligned}
$$

Finally:

$$
\begin{aligned}
f^2(p^2) &= \frac{1}{6} N_c (4\pi)^{-2} \int_0^1 dx\; 2x(1-x) p^2 \log \left\{ \frac{m_b^2 - x(1-x)p^2}{m_t^2 - x(1-x)p^2} \right\} \\
&+ \frac{1}{2} N_c m_t^2 (4\pi)^{-2} \int_0^1 dx\; \log \left\{ \Lambda^2/(m_t^2 - x(1-x)p^2) \right\} \\
&+ \frac{1}{2} N_c m_b^2 (4\pi)^{-2} \int_0^1 dx\; \log \left\{ \Lambda^2/(m_b^2 - x(1-x)p^2) \right\} \qquad (23)
\end{aligned}
$$

Note that $f(p^2)$ ($\overline{f}(p^2)$) may be interpreted as the decay constant of the neutral (charged) Nambu–Goldstone mode.

At this stage of the approximation it is useful to note the quantitative prediction for m_t in terms of G_F. Eq. (14) combined with eq.(16) gives:

$$\overline{f}^2(0) = \frac{1}{4\sqrt{2}G_F} \approx N_c(4\pi)^{-2}\int_0^1 (1-x)m_t^2 \log\left\{\Lambda^2/((1-x)m_t^2)\right\}$$
$$\approx \frac{1}{2}N_c(4\pi)^{-2}m_t^2\log\{\Lambda^2/m_t^2\} \tag{24}$$

For example, with $\Lambda = 10^{15}$ GeV one finds $m_t \approx 165$ GeV. To what extent is this an accurate prediction for m_t? For one, it is valid only in leading order of $1/N_c$ with $g_3 = 0$. This result, moreover, neglects the full dynamical effects of gauge bosons and the composite Higgs boson, which should be included in the renormalization group running below the scale Λ. We note that this result is substantially less than our full Standard Model result as obtained in Section IV.

We also see that the gauge couplings are subject to logarithmic evolution between the scales Λ and M_W. We may write the low energy gauge coupling constants:

$$\frac{1}{g_2^2(0)} = \frac{1}{g_2^2} + \frac{1}{6}N_c(4\pi)^{-2}\left\{\frac{4}{3}\log\left\{\Lambda^2/m_t^2\right\} + \frac{2}{3}\log\left\{\Lambda^2/m_b^2\right\}\right\} \tag{25}$$

and:

$$\frac{1}{g_1^2(0)} = \frac{1}{g_1^2} + \frac{1}{6}N_c(4\pi)^{-2}\left\{\frac{20}{9}\log\left\{\Lambda^2/m_t^2\right\} + \frac{2}{9}\log\left\{\Lambda^2/m_b^2\right\}\right\} \tag{26}$$

We also have the running of $\overline{g}_2$ from the W boson propagator:

$$\frac{1}{\overline{g}_2^2(0)} = \frac{1}{g_2^2} + N_c(4\pi)^{-2}\int_0^1 dx\; 2x(1-x)\log\left\{\Lambda^2/(xm_b^2 + (1-x)m_t^2)\right\} \tag{27}$$

The *high energy* renormalization group running of g_2 and $\overline{g}_2$ implied by the net coefficients of the $\log\Lambda$ in eq.(25) and eq.(27) is identical. Thus the high energy running in the unbroken phase corresponding to momenta $p^2 \gg m_t^2$ will be consistently that given by a single $SU(2)$ gauge coupling constant. Moreover, the high energy running of g_2 is consistent with a single generation quark–doublet contribution to the usual β–function:

$$16\pi^2\frac{\partial}{\partial\ln\mu}g_2 = \left\{-\frac{22}{3} + \frac{N_c}{3}n_q + \frac{1}{3}n_l\right\}g_2^3 \tag{28}$$

where n_q (n_l) is the number of quark (lepton) doublets. Thus, the coefficient of $\log\Lambda$ in eq.(25) or eq.(27) corresponds to $n_q = 1$ in the second term on the *rhs* of eq.(28), neglecting the first and third terms in the fermion bubble approximation.

Similarly, the high energy running of g_1 may be read off from eq.(26) and again is consistent with a single quark doublet contribution to the usual renormalization group equation:

$$16\pi^2\frac{\partial}{\partial\ln\mu}g_1 = \left\{\frac{11}{27}N_c n_q + n_l\right\}g_1^3 \tag{29}$$

The fact that we obtain the normal renormalization group running of these coupling constants from the single iso–doublet of quarks (neglecting all other contributions, such as gauge boson loops) indicates that *the low energy effective Lagrangian at this order is just the standard model.* The further renormalization effects below the scale m_t are radiative corrections that show up at low energy, *e.g.*, neutrino scattering for $Q^2 \ll M_W^2$. These involve, essentially, the extrapolation from the on–shell W and Z masses to the low energy measured $\sin^2\theta_W$ and G_F. Does the model lead to new effects here? We see, for example, that:

$$f^2(0) = \frac{1}{2}N_c(4\pi)^{-2}\left\{m_t^2\log\left\{\Lambda^2/m_t^2\right\} + m_b^2\log\left\{\Lambda^2/m_b^2\right\}\right\} \tag{30}$$

and:

$$\begin{aligned}\overline{f}^2(0) &= N_c(4\pi)^{-2}\int_0^1 dx\ \left\{xm_b^2 + (1-x)m_t^2\right\}\log\left\{\Lambda^2/(xm_b^2+(1-x)m_t^2)\right\} \\ &= f^2(0) + \frac{N_c}{4}(4\pi)^{-2}\left\{m_t^2 + m_b^2 - \frac{2m_t^2m_b^2}{m_b^2-m_t^2}\log(m_b^2/m_t^2)\right\}\end{aligned} \tag{31}$$

Veltman's ρ parameter is just $\rho = \overline{f}^2(0)/f^2(0)$ and thus the difference in $\overline{f}(0)^2$ and $f(0)^2$ is just the usual correction to the ρ–parameter due to weak isospin breaking effects and arises as a radiative correction to many physical processes. There are thus no additional corrections associated with the dynamical symmetry breaking mechanism beyond the usual standard model results. This is analogous to well known results of Carter and Pagels [21] for other dynamical symmetry breaking schemes such as technicolor.

This completes the discussion of the theory in the fermion bubble approximation. We turn now to an equivalent, but more transparent and ultimately more powerful discussion using the renormalization group and effective Lagrangians.

3. Low Energy Effective Lagrangian

3.1 Induced Higgs Scalar

In the previous section we derived the low energy effects of dynamical symmetry breaking provided by a sufficiently attractive four–fermion interaction involving the top quark as defined in eq.(1). We considered a model based on a conventional sum of the fermion bubble diagrams associated with the leading large–N_c limit with $g_3 = 0$. This simple model generates dynamical masses for the top quark and gauge bosons of the standard model, as well as a bound state corresponding to the usual physical Higgs scalar of mass $2m_t$. The fermion bubbles yield their conventional contribution to the running of the gauge coupling constants and the explicit cut–off dependence can be absorbed by appropriate renormalization of these couplings. The effective

Higgs vacuum expectation value, $\propto \overline{f}(0)$, has the normal isospin structure related to the ρ parameter but remains sensitive to the cut–off Λ as its dependence cannot be absorbed by renormalization. Our calculations imply that the effective low–energy dynamics is, in fact, just the usual standard model with certain constraints on the fundamental parameters of the theory.

We can see explicitly the connection with the standard model by using a Yukawa form of the four–fermion interactions as defined at the cut–off scale Λ, through the help of a static, auxiliary Higgs field, H [22]. We can rewrite eq.(1) as:

$$\mathcal{L} = \mathcal{L}_{kinetic} + (\overline{\Psi}_L t_R H + h.c.) - m_0^2 H^\dagger H \tag{32}$$

If we integrate out the field H we produce the four–fermion vertex as an induced interaction with $G = 1/m_0^2$. Note that only nontachyonic $m_0^2 > 0$ implies an attractive interaction and allows the factorization in this form.

Eq.(32) is the effective Lagrangian on a scale Λ. To obtain the effective Lagrangian on a scale $\mu < \Lambda$ in the fermion bubble approximation we integrate out the fermion field components on scale $\mu \to \Lambda$ as in Fig.(4):

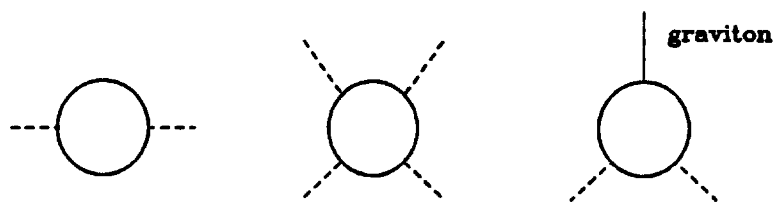

Figure (4): Block–spin renormalization group including only fermion loops.

The full induced effective Lagrangian at the scale μ then takes the form:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{kinetic} + \overline{\Psi}_L t_R H + h.c. + \Delta\mathcal{L}_{gauge} \\ &\quad + Z_H |D_\nu H|^2 - m_H^2 H^\dagger H - \frac{\lambda_0}{2}(H^\dagger H)^2 - \xi_0 R H^\dagger H \end{aligned} \tag{33}$$

where D_ν is the gauge covariant derivative and all loops are now to be defined with respect to a low energy scale μ. Here $\Delta\mathcal{L}_{gauge}$ contains the fermion loop contribution to the renormalization of the gauge coupling constants. We include an induced non-minimal coupling of the Higgs boson to gravity, ξ, [10]. A direct evaluation of the induced parameters in the Lagrangian gives as in Fig.(4):

$$\begin{aligned} Z_H &= \frac{N_c}{(4\pi)^2}\log(\Lambda^2/\mu^2); \qquad m_H^2 = m_0^2 - \frac{2N_c}{(4\pi)^2}(\Lambda^2 - \mu^2) \\ \lambda_0 &= \frac{2N_c}{(4\pi)^2}\log(\Lambda^2/\mu^2); \qquad \xi_0 = \frac{1}{6}\frac{N_c}{(4\pi)^2}\log(\Lambda^2/\mu^2). \end{aligned} \tag{34}$$

The Lagrangian of eq.(33) is, apart from normalization, exactly the same as the usual low energy standard model, except that the induced parameters, Z_H and λ_0, and ξ_0 are determined. Note that they all have an explicit dependence upon Λ, vanishing when $\mu \to \Lambda$.

We emphasize that *the effective theory applies in either the broken or unbroken phases.* The broken phase is selected by demanding that $m_H^2 < 0$ for scales $\mu \ll \Lambda$, thus requiring that $m_0^2 - 2N_c\Lambda^2/16\pi^2 < 0$. *This is equivalent to tuning the gap equation to produce the low energy dynamical symmetry breaking, i.e.,* $G > G_c = 8\pi^2/N_c\Lambda^2$ since $G = 1/m_0^2$. On the other hand, for positive m_H^2 as $\mu \to 0$ the theory remains unbroken (this is equivalent to a subcritical four–fermion coupling constant, $G \leq G_c$) and a massive Higgs boson doublet remains in the spectrum as a composite state.

Let us bring the effective Lagrangian of eq.(33) into a conventionally normalized form:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{kinetic} + g_t \overline{\Psi}_L t_R H + h.c. + \Delta\mathcal{L}_{gauge} \\ &+ |D_\nu H|^2 - \widetilde{m}_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 - \xi R H^\dagger H \end{aligned} \tag{35}$$

by rescaling the field $H \to H/\sqrt{Z_H}$. We then find:

$$\begin{aligned} g_t^2 &= 1/Z_H = 16\pi^2/N_c \log(\Lambda^2/\mu^2) \\ \widetilde{m}_H^2 &= m_H^2/Z_H \\ \lambda &= \lambda_0/Z_H^2 = 32\pi^2/N_c \log(\Lambda^2/\mu^2) \\ \xi &= \xi_0/Z_H = 1/6 \end{aligned} \tag{36}$$

These are the physically normalized coupling constants, and after fine–tuning the low energy value of $\widetilde{m}_H^2$ to obtain the spontaneously broken phase, the remaining predictions of the model are contained entirely in g_t, λ (and ξ) as we will see below. The compositeness of the Higgs boson essentially implies the results that g_t and λ become singular as $\mu \to \Lambda$ (while ξ remains constant and equal to its conformal value of 1/6). We will refer to these as the "compositeness conditions."

It is, however, instructive to see that these results are easily recovered directly from the conventional, differential renormalization group equations, supplemented with the compositeness conditions as boundary conditions. We utilize the partial β-functions which reflect only the presence of fermion loops:

$$16\pi^2 \frac{\partial}{\partial \ln \mu} g_t = N_c g_t{}^3 \tag{37}$$

$$16\pi^2 \frac{\partial}{\partial \ln \mu} \lambda = (-4N_c g_t^4 + 4N_c g_t^2 \lambda) \tag{38}$$

Solving eq.(37) gives:

$$\frac{1}{g_t^2(\mu)} - \frac{1}{g_t^2(\Lambda)} = \frac{N_c}{16\pi^2} \ln(\Lambda^2/\mu^2) \tag{39}$$

If we now use the boundary condition, $1/g_t^2(\Lambda) = 0$ we see that we recover eq.(36) for g_t^2. Eq.(38) may then be solved by hypothesizing an anzatz of the form $\lambda = cg_t^2$. Substituting into eq.(38) one finds:

$$16\pi^2 \frac{\partial}{\partial \ln \mu} g_t = \frac{1}{2c}(4c-4)N_c {g_t}^3 \tag{40}$$

and demand that this must be consistent with eq.(37). Thus one finds $c = 2$ and:

$$\frac{1}{\lambda(\mu)} - \frac{1}{\lambda(\Lambda)} = \frac{N_c}{32\pi^2}\ln(\Lambda^2/\mu^2) \tag{41}$$

and again $1/\lambda(\Lambda) = 0$ leads to the result of eq.(36) for λ.

Now, to obtain the usual phenomenological results of the NJL model we examine the low energy Higgs potential from eq.(35) with $\mu = m_t$:

$$V(H) = -\widetilde{m}_H^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2 - (g_t \overline{\Psi}_L t_R H + h.c.) \tag{42}$$

Let us assume that $\widetilde{m}_H^2 < 0$ so the neutral Higgs field develops a VEV: $Re(H^0) = v + \phi/\sqrt{2}$. In the standard model we assume that v has been fine–tuned to the physical value of $v^2 = 1/2\sqrt{2}G_F = (175)^2$ GeV.

Therefore we find the top mass:

$$m_t = g_t v; \tag{43}$$

and the ϕ mass:

$$m_\phi^2 = 2v^2\lambda \tag{44}$$

and so:

$$m_\phi^2/m_t^2 = 2\lambda/g_t^2 = 4 \tag{45}$$

where we use the explicit solutions eq.(39) and eq.(41) for $\lambda/g_t^2 = 2$. This is the familar NJL result, $m_\phi = 2m_t$. Moreover, we have:

$$v^2 = m_t^2/g_t^2 = m_t^2 \frac{N_c}{16\pi^2}\ln(\Lambda^2/m_t^2) = \frac{1}{2\sqrt{2}G_F} \tag{46}$$

which is equivalent to the prediction obtained from a direct fermion bubble approximation computation of the decay constant. We have seen that the RG directly and simply reproduces the result of a "brute force" summation of fermion bubbles.

It is also amusing to study the result $\xi = 1/6$ in the differential renormalization group. The RG equation for $\xi = \xi_0/Z_H$ can be derived by considering,

$$\frac{\partial}{\partial \ln \mu}\xi = \frac{\partial(\xi_0)}{\partial \ln \mu}/Z_H - \xi_0 \frac{\partial(Z_H)}{\partial \ln \mu}/Z_H^2. \tag{47}$$

In the fermion bubble approximation, we know from eq.(36) that $g_t^2 = 1/Z_H$ and hence,

$$16\pi^2 \frac{\partial}{\partial \ln \mu} g_t^2 = 2N g_t^4. \tag{48}$$

Eq.(47) then becomes,

$$16\pi^2 \frac{\partial}{\partial \ln \mu} \xi = -\frac{N}{3} g_t^2 + 2N\xi g_t^2, \tag{49}$$

and the solution for the curvature coupling parameter,

$$\xi(\mu) = 1/6, \tag{50}$$

is a constant for all scales. More generally, as one descends toward the infrared, $\xi = 1/6$ is an attractive fixed point. Therefore, no matter what is the initial value for ξ at the large scale Λ, given enough RG running time ξ will eventually reach 1/6 for small μ. Of course, the RG running only occurs for scales $\mu > m_H$.

We thus find in the usual fermion bubble approximation that ϕ is conformally coupled to gravity, even though scale breaking dynamics exists at high energies Λ. Moreover, $\xi = 1/6$ is an attractive renormalization group fixed point in the infrared in this approximation. This implies that, even if there are corrections to $\xi = 1/6$ from irrelevant operators at Λ, the observed low energy coupling is quickly attracted to a physical or "observed" value of $\xi = 1/6$ as one evolves into the infrared. Remarkably, even when more physics is included beyond the simple fermion bubble approximation by using the full one–loop renormalization group, this result persists. This is closely related to previous results which analyzed the RG behavior of ξ for large curvature [23].

3.2 Ladder QCD

We can now take a step closer to the full theory by including the effects of gluons in the RG equations [24]. This analysis illustrates again the power of the renormalization group in reproducing the results of this approximation. This is not yet a full improvement over the fermion bubble results, since obviously the ladder QCD calculations omit the propagating Higgs boson, as well as electroweak effects which are included in the full RG equations below.

We now have, including only the effects of fermion loops and gluons in loops:

$$16\pi^2 \frac{\partial}{\partial \ln \mu} g_t = N_c {g_t}^3 - (N_c^2 - 1) g_3^2 g_t; \tag{51}$$

$$16\pi^2 \frac{\partial}{\partial \ln \mu} g_3 = -(11 - 2n_f/3) g_3^3; \tag{52}$$

$$16\pi^2 \frac{\partial}{\partial \ln \mu} \lambda = (-4N_c g_t^4 + 4N_c g_t^2 \lambda). \tag{53}$$

Notice the additional QCD term in eq.(51) in comparison to eq.(37). The equation for λ is unchanged at this one–loop (leading log) order. Again, the UV boundary conditions on the theory are as before, $g_t \to \infty$ and $\lambda \to \infty$ as $\mu \to \Lambda$. It is most convenient to obtain these results numerically, and they are indicated in Table I for various values of Λ. We see for the first time the appearance of the nontrivial RG infrared fixed point for g_t at this stage [13].

4. Fully Improved Renormalization Group Solution

4.1 Infrared Fixed Points

To obtain the full renormalization group improvement over the Nambu–Jona-Lasinio model we may utilize the compositeness boundary conditions on g_t and λ and the *full* β-functions (we'll neglect light quark masses and mixings) of the standard model. To one–loop order we have:

$$16\pi^2 \frac{\partial}{\partial \ln \mu} g_t = \left((N_c + \frac{3}{2}) {g_t}^2 - (N_c^2 - 1) {g_3}^2 - \frac{9}{4} {g_2}^2 - \frac{17}{12} {g_1}^2 \right) g_t \qquad (54)$$

and, for the gauge couplings:

$$16\pi^2 \frac{\partial}{\partial \ln \mu} g_i = -c_i \, {g_i}^3 \qquad (55)$$

with

$$c_1 = -\frac{1}{6} - \frac{20}{9} N_g; \quad c_2 = \frac{43}{6} - \frac{4}{3} N_g; \quad c_3 = 11 - \frac{4}{3} N_g \qquad (56)$$

where N_g is the number of generations and $t = \ln \mu$. The principle differences in eq.(37) relative to eq.(54) are: (i) inclusion of the propagating dynamically generated Higgs boson in loops (the additional 3/2 in the coefficient of g_t^3) and (ii) the inclusion of electroweak effects.

The precise value of the top quark mass is determined by running $g_t(\mu)$ down from a given compositeness scale Λ at which $g_t(\Lambda) = \infty$, or in practice, is large. The evolution ends when the mass–shell condition $g_t(m_t)v = m_t$ is reached. We will not discuss possible low energy corrections associated with the extrapolation of the symmetric three–point function to a zero–momentum Higgs line.

The nonlinearity of eq.(54) focuses a wide range of initial values into a small range of final low energy results [12, 13]. The solution for $m_{quark} = g_t(\mu)v$ is shown in Fig.(5) for $\Lambda = 10^{15}$ GeV (case A) and $\Lambda = 10^{19}$ GeV (case B) respectively. This is a "quasi" infrared fixed point, which would be an exact fixed point if g_3 were constant. The interpretation of the fixed point behavior is that of [13]. The fixed point is a reflection of approximate scale invariance (vanishing β function) of the theory as we tune the gap equation to produce $m_t \ll \Lambda$. The scale invariance is explicitly broken by Λ_{QCD}.

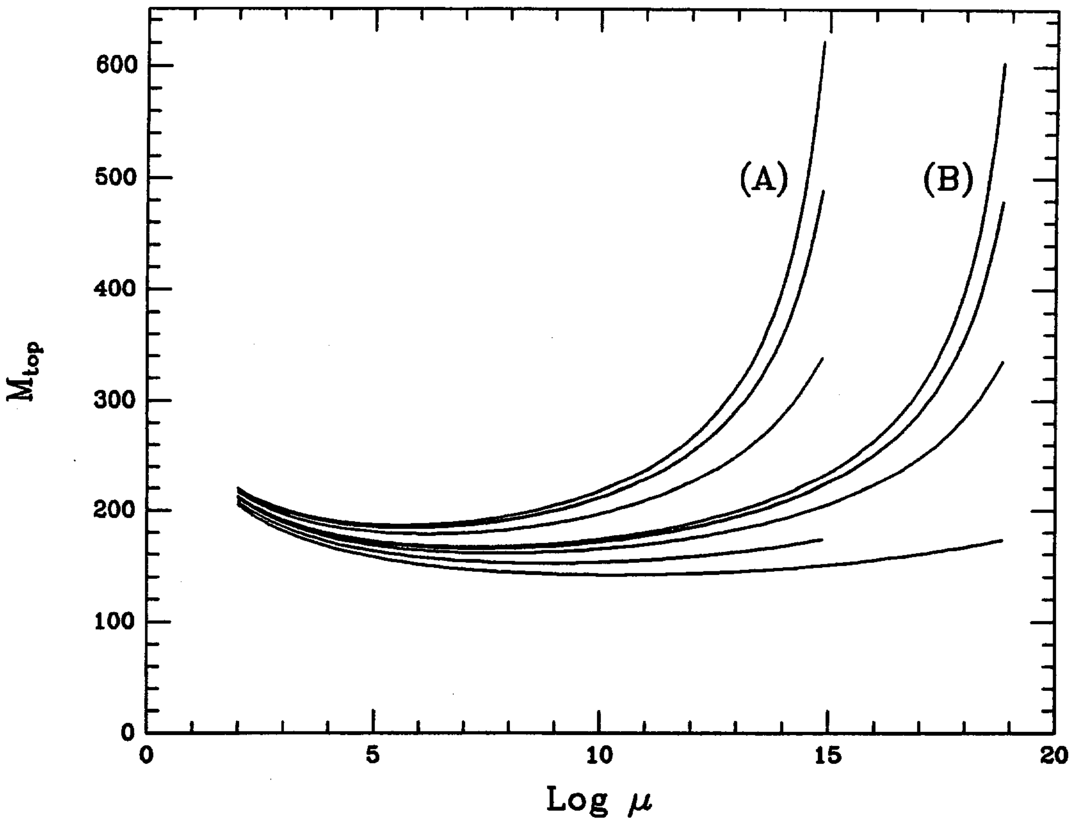

Figure (5): Full RG trajectories as a function of scale μ. (A) $\Lambda = 10^{15}$ GeV; (B) $\Lambda = 10^{19}$ GeV. The composite trajectories diverge at the corresponding value of Λ. The predicted m_{quark} is controlled by the quasi–infrared fixed point and is very insensitive to Λ [9], [13].

The quasi–fixed point behavior implies that m_t is determined up to $O(\ln\ln\Lambda/m_t)$ sensitivity to Λ. In Table I we give the resulting physical m_{top} obtained by a numerical solution of the renormalization group equations as a function of Λ. Note the sensitivity to Λ is reduced when the nontrivial IR fixed point is present.

The Higgs boson mass will likewise be determined by the evolution of λ now given by:

$$16\pi^2 \frac{\partial}{\partial \ln\mu}\lambda = 12(\lambda^2 + (g_t^2 - A)\lambda + B - g_t^4) \tag{57}$$

where:

$$A = \frac{1}{4}g_1^2 + \frac{3}{4}g_2^2; \quad B = \frac{1}{16}g_1^4 + \frac{1}{8}g_1^2 g_2^2 + \frac{3}{16}g_2^4 \tag{58}$$

There are now significant modifications in eq.(57) relative to eq.(38) due to the inclusion of virtual Higgs propagation (the first term on the *rhs*) and electroweak interactions. As in the case of g_t, we evolve $\lambda(\mu)$ from the compositeness scale Λ down to the weak scale with the compositeness boundary condition, $\lambda(\mu \to \Lambda) \to \infty$. The joint evolution of g_t and λ to the RG fixed point is shown in Fig.(6), and m_H is given in Table I including the full RG effects.

Λ (GeV)	10^{19}	10^{15}	10^{11}	10^{7}	10^{5}
m_t (GeV); Fermion Bubble[a]	144	165	200	277	380
m_t (GeV); Planar QCD[a]	245	262	288	349	432
m_t (GeV); Full RG[b]	218	229	248	293	360
m_H (GeV); Full RG[b]	239	256	285	354	455

Table I: Predicted m_{top} in three levels of increasingly better approximation as described in the text. "Fermion Bubble" refers only to the inclusion of fermion loops, equivalent to the conventional Nambu–Jona-Lasinio analysis, in which case $m_H = 2m_t$. "Planar QCD" includes additional effects of internal gluon lines. All effects, including internal Higgs lines and electroweak corrections, are incorporated in the "Full RG" lines, and we include the m_H results. *Notice that the full renormalization effects cause* $m_H \neq 2m_t$. Results ([a]) are from Mahanta and Barrios, [24] and ([b]) are from [9].

4.2 Sensitivity to Irrelevant Operators

The action of the effective fixed point appears to make the top quark and Higgs boson mass predictions largely insensitive to the precise values of the coupling constant close to the scale Λ [12, 13]. Indeed, there may be real physical effects which modify the high energy boundary conditions. These effects may be due to the presence of normally irrelevant, higher dimension operators, or higher order corrections to the four fermion interactions at the scale Λ which are not already contained in the full renormalization group analysis. The higher dimension operators were first considered by Suzuki [14], and his analysis has been generalized by Hasenfratz *et al.* , [15]. How sensitive is the infrared physics to these model–dependent effects at high energy? We will show that these "Suzuki effects" are in fact rather small for a reasonable range of the coefficients of these new operators.

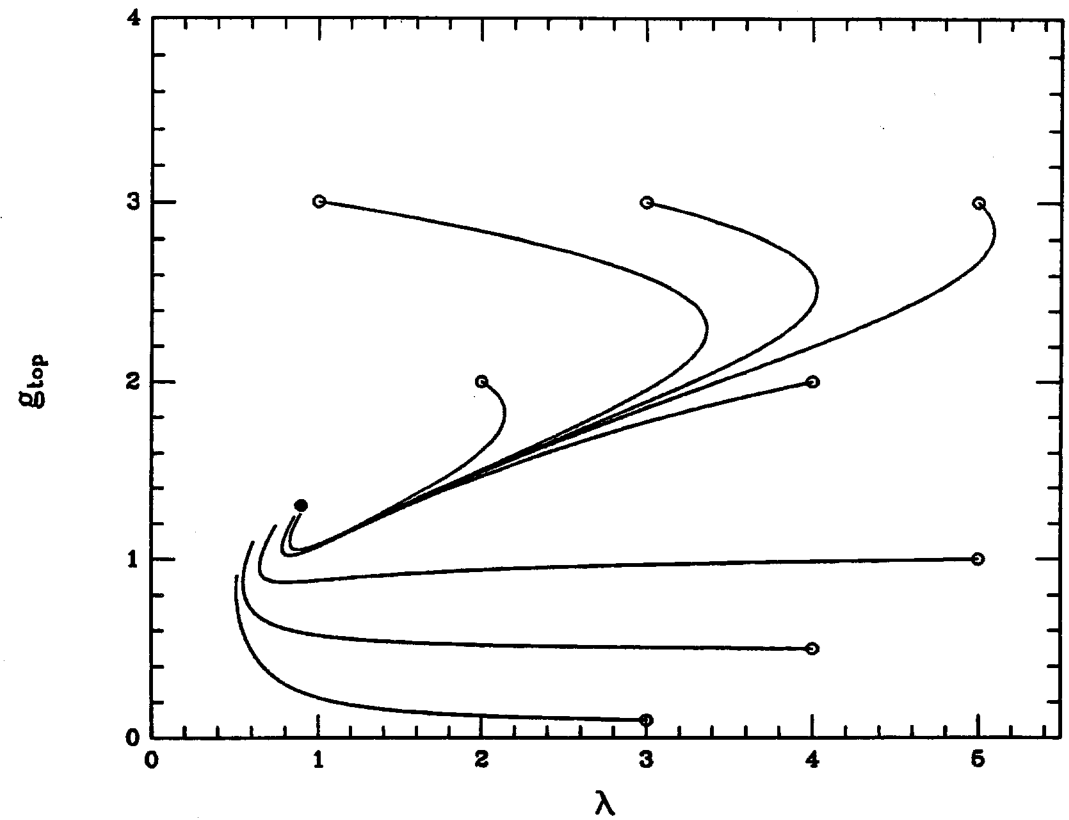

Figure (6): Full RG trajectories showing joint evolution of g_t and λ for various initial values. Compositeness corresponds to large initial g_t and λ, and these are attracted toward the nontrivial IR fixed point (solid circle).

We take our starting point Lagrangian, eq.(1), to be modified as

$$L = L_{kinetic} + G\left(\overline{\Psi}_L^{ia} t_{Ra} + \frac{\chi}{\Lambda^2}(D_\mu \overline{\Psi}_L^{ia})(D^\mu t_{Ra})\right)\left(\bar{t}_R^b \Psi_{Lib} + \frac{\chi}{\Lambda^2}(D_\mu \bar{t}_R^b)(D^\mu \Psi_{ibL})\right) \quad (59)$$

hence eq.(32) is similarly modified:

$$L = L_{kinetic} + \left((\overline{\Psi}_L^{ia} t_{Ra} + \frac{\chi}{\Lambda^2}(D_\mu \overline{\Psi}_L^{ia})(D^\mu t_{Ra}))H_i + h.c.\right) - M_0^2 H^\dagger H \quad (60)$$

Now, we perform the block–spin RG transformation as in section 2.1. we obtain the low energy effective Lagrangian in analogy with eq.(22):

$$\begin{aligned} L \;&=\; L_{kinetic} + \left((\overline{\Psi}_L^{ia} t_{Ra} H_i + h.c.\right) \\ &\quad + Z_H |D_\mu H|^2 - M_\mu^2 H^\dagger H - \frac{\lambda_0}{2}(H^\dagger H)^2 + O(1/\Lambda^2)... \end{aligned} \quad (61)$$

where now the parameters transform as:

$$Z_H = \frac{N_c}{8\pi^2}\left(\ln(\Lambda/\mu) - \chi + \chi^2/8\right) \tag{62}$$

$$\lambda_0 = \frac{N_c}{4\pi^2}\left(\ln(\Lambda/\mu) - 2\chi + \frac{3}{2}\chi^2 - \frac{2}{3}\chi^3 + \frac{1}{8}\chi^4\right) \tag{63}$$

and M_μ^2 has additive terms which we will fine–tune as above.

To obtain the low energy predictions the renormalizaton group equations are modifeid by physics near the scale Λ which depends upon χ. At scales far below Λ the usual renormalization group equations apply with modifications of the high energy boundary conditions. We incorporate these effects by using exact results for large N at scales $\mu \sim \Lambda$ as given in eqs.(62, 63), but then use the full RG analysis at lower energies where the higher dimension operators decouple.

The following procedure has been adopted to explore the sensitivity to χ: (i) from $\mu = \Lambda$ to $\mu = \mu^* = \Lambda/5$ we use eq.(62) and eq.(63) directly to evolve Z_H and λ_0; (ii) from $\mu = \mu^*$ to $\mu = m_t$ we use the RG equations. The sensitivity of the low energy predictions is shown in Fig.(6) for the three cases: (1) fermion bubble approximation; (2) ladder QCD; and (3) full standard model. The most sensitive case is that of fermion loop approximation since we see that there is no real nontrivial fixed point to the RG equations in that case. For a wide range of χ the planar QCD and full standard model predictions are very insensitive owing to the nontrivial fixed point for large g_t which is rapidly approached.

Recently Hasenfratz, *et al.* [15] have generalized the Suzuki analysis by including a complete set of higher dimension four fermion interactions. They show that these interactions can cause independent, finite shifts to the values of Z_H and λ_0. With arbitrarily large coefficients of the higher derivative interactions they claim that any physical prediction for m_{top} and m_{Higgs} can be obtained. They conclude that the top condensate theory is unpredictive and that a very light top quark is therefore consistent with the electroweak symmetry breaking coming only from short range interactions of the elementary fermions.

The results of Hasenfratz *et al.* are only demonstrated in the fermion bubble approximation, in which they are true mathematically, but require unphysically large values of the coefficients of the new operators for their conclusions to apply. They require that the finite corrections at the high energy scale dominate the large logarithm arising from the evolution to the weak scale. Moreover, the focusing effects of the infrared fixed points are ignored by considering only the fermion bubble approximation, and these effects will further stabilize the predictions as we have seen previously. As we have demonstrated, the actual results are, in fact, very insensitive to these corrections if the coefficients of the new operators are $O(1)$.

The ultimate issue of the size of the residual corrections to the leading four–fermion operator resides in the nature of the parent theory, which is valid on scales $\gg \Lambda$. In a full, realistic theory in which the interactions at the scale Λ are generated dynamically we can hope to compute χ. We turn to this possibility in the next section. In the following we consider one such model, and indeed it is found in ladder approximation that there are residual corrections, but these are very small (we find $\chi \sim 0.1$ rather than $\chi \sim 10$ which is required for any significant impact on the low energy predictions) We refer the reader to ref.[16] for the details of this estimate.

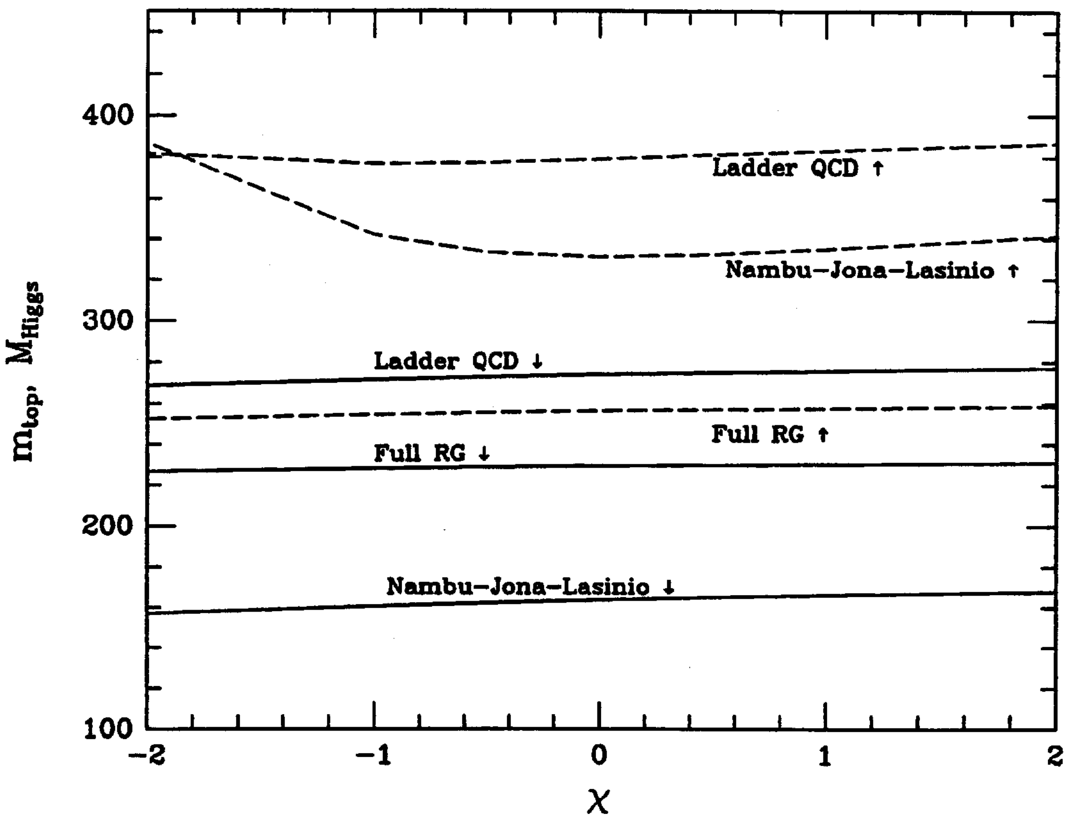

Figure (7): Sensitivity of predicted m_{top} (solid lines) and m_{Higgs} (dashed lines) to $d = 6$ operator coefficient χ.

5. Top-Color: A Gauge Theory that makes a Top Condensate

The interaction introduced in eq.(1) is clearly only an effective description of a more primitive theory. A Fierz rearrangement of the interaction leads to:

$$(\overline{\psi}^a_L t_{Ra})_i(\bar{t}_{Rb}\psi^b)^i \to -(\overline{\psi}_{iL}\gamma_\mu \frac{\lambda^A}{2}\psi^i_L)(\bar{t}_R\gamma^\mu\frac{\lambda^A}{2}t_R) + O(1/N) \tag{64}$$

where $N = 3$ is the number of colors. This form strongly suggests a new gauge theory leading to a current–current form of the effective Lagrangian. We further note that: (i) this gauge theory must be broken at a scale of order Λ; (ii) it is strongly coupled at the breaking scale; (iii) it involves the color degrees of freedom of the top quark (or fourth generation fermions) in a manner analogous to QCD. The relevant models will involve the embedding of QCD into some large group G which is sensitive to the flavor structure of the standard model.

Let us construct a minimal version of such a theory. We presently seek a gauge interaction which leads to a term as in eq.(2) but which, like minimal technicolor, will leave the light quarks and leptons massless. A subsequent extension of the theory is required to give masses and mixing angles to light fermions, and we do not address this issue at present. Therefore, consider an extension of the standard model such that at scales $\mu \gg \Lambda$, we have $U(1)\times SU(2)_L \times SU(3)_1 \times SU(3)_2$. We assign the usual light quark and lepton fields to representations under $(SU(2)_L, SU(3)_1, SU(3)_2)$ such that they transform as singlets under the new $SU(3)_2$, as follows:

$$\begin{array}{rrrcl}
 & (u,\ d)_L; & (c,\ s)_L & \to & (2,3,1) \\
(\nu_e,\ e)_L; & (\nu_\mu,\ \mu)_L; & (\nu_\tau,\ \tau)_L & \to & (2,1,1) \\
 & & u_R,\ d_R,\ c_R,\ s_R,\ b_R & \to & (1,3,1) \\
 & & e_R,\ \mu_R,\ \tau_R,\ (\nu_{iR}) & \to & (1,1,1)
\end{array} \tag{65}$$

while the top quark is a singlet under the first $SU(3)_1$ group:

$$(t,\ b)_L \to\ (2,1,3); \qquad t_R \to\ (1,1,3) \tag{66}$$

This assignment is not anomaly free, and we can minimally realize all anomaly cancellations provided we introduce the following electroweak singlet quarks:

$$Q_R \to (1,1,3); \qquad Q_L \to (1,3,1) \tag{67}$$

Both Q_R and Q_L have weak hypercharge $Y = -2/3$, hence electric charge $Q = -1/3$.

Since we wish to break the symmetry $SU(3)_1 \times SU(3)_2 \to SU(3)_c$ at the scale Λ, we introduce a scalar (Higgs) field $\Phi^a_{b'}$ which transforms as $(1,3,\overline{3})$. By the simplest choice of the Φ potential a VEV develops of the form: $\langle\Phi\rangle = \mathrm{diag}(\Lambda,\Lambda,\Lambda)$. This VEV breaks $SU(3)_1 \times SU(3)_2$ to a massless gauge group $SU(3)_c$ with gluons, A^A_μ and a residual global $SU(3)'$ with degenerate, massive gauge bosons ("colorons") B^A_μ.

Q must be given a large enough Dirac mass, $\gtrsim \Lambda$, so that it does not further influence the dynamical symmetry breaking. We invoke a large Higgs–Yukawa coupling of the Φ field to the combination $\overline{Q}_L Q_R$. Thus, if we take:

$$\kappa \Phi_a^{b'} \overline{Q}_L^a Q_{Rb'} + h.c. \tag{68}$$

then Q receives a mass of $\kappa\Lambda$. A lower bound on κ will be estimated below such that the Q field may be approximated as having decoupled at the scale Λ. It should be noted, however, that with the given the quantum numbers of $\overline{Q}Q$ there is an intriguing possibility that in extensions of this scheme the $\left\langle \overline{Q}Q \right\rangle$ condensate may form dynamically breaking $SU(3)_1 \times SU(3)_2 \to SU(3)_c$, so that an explicit Φ field may not be required. For example, if we assign instead $(c,\ s)_L \to (2,1,3)$, we find that anomaly cancellation requires the Q be a triplet with $Y = 0$! Gauging this triplet with yet another $SU(3)_3$ allows a QCD–like chiral condensate of the form $\left\langle \overline{Q}Q \right\rangle$ which is $(1, \overline{3}, 3)$, and the symmetry breaks as described here. This model leads to a low energy two–Higgs doublet scheme.

Returning to the simple example, let the coupling constants of $SU(3)_1 \times SU(3)_2$ be respectively h_1 and h_2. Then the gluon (A_μ^A) and coloron (B_μ^A) fields are defined by

$$\begin{aligned} A_{1\mu}^A &= \cos\theta A_\mu^A - \sin\theta B_\mu^A \\ A_{2\mu}^A &= \sin\theta A_\mu^A + \cos\theta B_\mu^A \end{aligned} \tag{69}$$

where:

$$h_1 \cos\theta = g_3; \qquad h_2 \sin\theta = g_3; \tag{70}$$

and thus:

$$\tan\theta = h_1/h_2; \qquad \frac{1}{g_3^2} = \frac{1}{h_1^2} + \frac{1}{h_2^2} \tag{71}$$

where g_3 is the QCD coupling constant at Λ. In what follows we envision $h_2 > h_1$ and thus $\tan\theta < 1$ to select the top quark direction for condensation. The mass of the degenerate octet of colorons is given by:

$$M_B = \left(\sqrt{h_1^2 + h_2^2}\right)\Lambda = \left(\frac{2g_3}{\sin 2\theta}\right)\Lambda \tag{72}$$

The $SU(3)_c$ current will be the usual QCD current for all quarks while the $SU(3)'$ current (multiplied by its coupling strength) takes the form:

$$\begin{aligned} h_\mu^A \;=\; & g_3 \cot\theta \left(\bar{t}\gamma_\mu \frac{\lambda^A}{2} t + \bar{b}_L \gamma_\mu \frac{\lambda^A}{2} b_L + \overline{Q}_R \gamma_\mu \frac{\lambda^A}{2} Q_R \right) \\ & + g_3 \tan\theta \left(\bar{b}_R \gamma_\mu \frac{\lambda^A}{2} b_R + \overline{Q}_L \gamma_\mu \frac{\lambda^A}{2} Q_L + \sum_i \bar{q}_i \gamma_\mu \frac{\lambda^A}{2} q_i \right) \end{aligned} \tag{73}$$

where the sum extends over all first and second generation quarks. If $h_2 \gg h_1$ the dominant coloron mediated interaction takes the form of eq.(1) provided we identify:

$$\frac{g^2}{\Lambda^2} = \frac{g_3^2 \cot^2 \theta}{M_B^2} = \frac{\cos^2 \theta}{\Lambda^2} \tag{74}$$

Let is now ask what condition on θ implies dynamical symmetry breaking through the formation of a top condensate. The scale at which the four fermion interaction softens to a gauge boson exchange is given by the mass of the coloron M_B, and we may treat the effective interaction as a four–fermion form at all scales $\mu \ll M_B$. Therefore, in the large–N approximation the gap equation can be written for the spontaneous formation of the top–condensate with a momentum cut–off taken to be $\sim M_B$ [9]:

$$m_t = m_t \frac{g_3^2 N \cot^2 \theta}{8\pi^2 M_B^2}\left[M_B^2 - m_t^2 \log(M_B^2/m_t^2)\right] \tag{75}$$

and the existence of the condensate implies:

$$\frac{g_3^2 N \cot^2 \theta}{8\pi^2} > 1 \qquad \text{or} \qquad \frac{N}{2\pi}\alpha_3(M_B)\cot^2\theta \geq 1 \tag{76}$$

where $\alpha_3 = g_3^2/4\pi$.

On scales below the M_B we expect that the analysis of ref.[9] holds. If $M_B \gg M_W$ then to have an acceptable top mass we must fine–tune θ so that $\frac{N}{2\pi}\alpha_3(M_B)\cot^2\theta \approx 1$ to a high precision. It is also crucial that the spectator Q be sufficiently heavy so that a $\overline{\psi}Q$ condensate *does not form* (the custodial $SU(2)_R$ leads to problems with extra unwanted Goldstone bosons and may ultimately break $U(1)_{EM}$). For a heavy fermion in the gap loop a sufficient condition that no breaking occur in this channel is:

$$\kappa > m_t/(M_B \log(M_B/m_t)) \tag{77}$$

provided the mixing angle is fine–tuned to produce the low mass top condensate. Essentially this condition insures that Q decouples and the associated quadratic divergence becomes $\Lambda^2 - M_Q^2$, and the interaction has insufficient strength to drive the condensate.

In related works, possible horizontal interactions have been considered by T. K. Kuo, *et al.* , and a $U(1)$ version of this scheme is being developed by R. Bonisch, and independently by M. Lindner and D. Ross [25]. We refer the reader to [16] for a discussion of the size of χ in these models, which is generally found to be small.

6. Supersymmetric Models of Top Condensates

We can construct a minimal supersymmetric extension of the standard model in which electroweak symmetry breakdown involves formation of condensates of the third generation of quarks and their supersymmetric partners. The top quark mass is then obtained as a function of the compositeness scale Λ and the soft supersymmetry breaking scale, Δ_S. When $\Lambda \simeq 10^{16}$ GeV, the characteristic top quark mass in this model is $140\ GeV \leq m_t \leq 195\ GeV$, a prediction that is only slightly dependent on the value of Δ_S. Supersymmetry provides a possible solution to the naturalness problem since above the SUSY breaking scale the quadratic divergences disappear, and hence no large fine tuning of the four Fermi coupling constants, or in general of the Higgs mass parameters, is required [17]. Hence the previous cut–off Λ^2 is replaced by Δ_S^2 which can, in principle, be near the weak scale.

The minimal supersymmetric extension of the composite-Higgs model was first studied in Ref.[17], in a simplified version in which only one of the scalar Higgs doublets acquires a vacuum expectation value, and hence, the bottom quark remains massless. The resulting infrared quasi–fixed point value of the top Yukawa coupling is slightly lower than in the case of the standard model. However, in a generalized scheme with two Higgs VEV's [18] in which the b quark acquires mass the top quark mass values can be significantly lower than those obtained in Ref.[17]. An important consequence of these radiative effects is to invalidate previous phenomenological constraints on the ratio R of the Higgs vacuum expectation values, and hence, lower values for the top quark mass are allowed within the model. Modifications to the top quark mass predictions induced by the appearance of a finite bottom quark Yukawa coupling is relevant.

6.2 Renormalization Group Flow of the Low Energy Parameters.

To describe the dynamics responsible for the top quark multiplet condensation, we shall consider an $SU(3) \times SU(2) \times U(1)$ invariant gauged supersymmetric Nambu-Jona-Lasinio model [17], [18], with explicit soft supersymmetry breaking terms. In this simplified model we ignore all quark and lepton Yukawa couplings except one associated with the top quark, since they are inessential for the qualitative description of the phenomena under study.

Written in terms of the two composite chiral Higgs superfields H_1 and H_2, the action of the gauged Nambu-Jona-Lasinio model at the scale Λ takes the form

$$
\begin{aligned}
\Gamma_\Lambda &= \Gamma_{YM} + \int dV \left[\overline{Q}e^{2V_Q}Q + T^C e^{-2V_T}\overline{T}^C + B^C e^{-2V_B}\overline{B}^C\right](1-\Delta^2\theta^2\overline{\theta}^2) \\
&+ \int dV \overline{H}_1 e^{2V_{H_1}} H_1 (1 - M_H^2\theta^2\overline{\theta}^2) \\
&- \int dS\, \epsilon_{ij}\left(m_0 H_1^i H_2^j(1+B_0\theta^2) - g_{T_0} H_2^j Q^i T^C(1+A_0\theta^2)\right) \\
&- \int d\overline{S}\, \epsilon_{ij}\left(m_0 \overline{H}_1^i \overline{H}_2^j(1+B_0\overline{\theta}^2) - g_{T_0} \overline{T}^C \overline{Q}^i \overline{H}_2^j(1+A_0\overline{\theta}^2)\right), \qquad (78)
\end{aligned}
$$

where $Q = \binom{T}{B}$ is the SU(2) doublet of top and bottom quark chiral superfield multiplets, T^C (B^C) is the SU(2) singlet charge conjugate top (bottom) quark chiral multiplet, and we have denoted the superspace integration measures $dV = d^4x d\theta^2 d\overline{\theta}^2$, $dS = d^4x d\theta^2$ and $d\overline{S} = d^4x d\overline{\theta}^2$ [26]. An equivalent form of the above action, only in terms of the fundamental quark chiral superfields, can be obtained by integrating out the static composite chiral superfields, or equivalently, by substituting in Eq.(78) the fields H_1 and H_2 in terms of their Euler-Lagrange equations. Γ_{YM} includes the usual supersymmetric gauge field kinetic and the supersymmetry breaking gaugino mass terms, while the quark and Higgs multiplets interact with the SU(3)×SU(2)×U(1) gauge fields via

$$\begin{aligned} V_Q &= g_3 G^a \frac{1}{2}\lambda^a + g_2 W^i \frac{1}{2}\sigma^i + \frac{1}{6} g_1 Y, \qquad & V_T = g_3 G^a \frac{1}{2}\lambda^a + \frac{2}{3} g_1 Y \\ V_B &= g_3 G^a \frac{1}{2}\lambda^a - \frac{1}{3} g_1 Y, \qquad & V_{H_1} = \frac{g_2}{2} W^i \sigma^i - \frac{1}{2} g_1 Y. \end{aligned} \tag{79}$$

We have included two soft supersymmetry breaking terms A_0 and B_0 which are proportional to the scalar trilinear and bilinear terms appearing in the superpotential. The gauged Nambu-Jona-Lasinio model depends only on $\delta = A_0 - B_0$, as can be easily verified by integrating out the chiral Higgs superfields. The inclusion of the δ induced terms in the low energy theory is essential in order to obtain nontrivial vacuum expectation values for both neutral scalar Higgs particles without inducing an unacceptably light axion [27]. Δ^2 and M_H^2 provide explicit soft supersymmetry breaking scalar mass terms.

It follows from the H_2 Euler-Lagrange equations that the scalar component of the H_1 chiral superfield and the quark superfields are related by

$$m_0 H_1 = g_{T_0} \widetilde{Q} \widetilde{T}^C, \tag{80}$$

where we have denoted by $\widetilde{Q}$ ($\widetilde{T}^C$) the scalar component of the chiral quark superfield Q (T^C). Since the soft supersymmetry breaking terms are thought to arise from an underlying supergravity [28], it is reasonable to assume that the higher dimension composite fields H_1 feel twice the breaking strength as do the individual $\widetilde{Q}$ or $\widetilde{T}^C$ fields. It is straightforward to prove that this is achieved when the H_1-explicit supersymmetry breaking mass term is given by $M_H^2 = 2\Delta^2 + \delta^2$.

In the presence of a condensate of top quark superfields, a dynamical mass for the top quark is generated. Its value may be determined in a self consistent way by using the Schwinger-Dyson equations in the bubble approximation. Note that, generalizing the expression for the superfield propagators derived in ref.[29], there is a left-right scalar quark propagator induced by the inclusion of the soft supersymmetry breaking term δ. The gap equation is:

$$G^{-1} = \frac{N_C \Delta^2}{16\pi^2} \left[\left(1 + \frac{2m_t^2 + \delta^2 \alpha}{2\Delta^2} \right) \ln \left(\frac{\Lambda^4}{(m_t^2 + \Delta^2)^2 - m_{QQ^C}^4} \right) - \frac{2m_t^2}{\Delta^2} \ln \left(\frac{\Lambda^2}{m_t^2} \right) \right]. \tag{81}$$

where $G = g_{T_0}^2/m_0^2$ and $\alpha = m_{QQ^C}^2/(\delta m_t)$ is given by

$$\alpha^{-1} = 1 + \frac{G M_H^2 N_C}{32\pi^2} \ln\left(\frac{\Lambda^4}{(m_t^2+\Delta^2)^2 - m_{QQ^C}^4}\right). \tag{82}$$

The logarithmic term in Eq.(82) comes from the interactions induced by the explicit scalar supersymmetry breaking mass term associated with the scalar field H_1. The gap equation takes a simpler form in the case in which this explicit mass term vanishes. In general, however, the critical value of the four Fermi coupling G may be obtained by solving the above two coupled equations. The logarithmic dependence on the compositeness scale Λ is a direct consequence of the supersymmetric nonrenormalization theorems [30]. *The usual quadratic dependence on Λ, appearing in the standard top-condensate models has been replaced by a mild quadratic dependence on the supersymmetry breaking scales Δ and δ.* Thus, as we have already mentioned, no fine tuning is necessary in this model.

In the scaling region, in which the four Fermi coupling constant is close to its critical value, a gauge invariant kinetic term for H_2 is induced at low energies. In the large N_C limit, it is given by [17]:

$$Z_{H_2} \int dV H_2 e^{2V_{H_2}} \overline{H}_2 (1 + A_0\theta^2 + A_0\overline{\theta}^2 + (2\Delta^2 + A_0^2)\theta^2\overline{\theta}^2), \tag{83}$$

where Z_{H_2} is the H_2 wavefunction renormalization constant, which at a normalization scale μ is given by

$$Z_{H_2} = \frac{g_{T_0}^2 N_C}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \tag{84}$$

and $V_{H_2} = -V_{H_1}$. When μ approaches Λ, Z_{H_2} tends to zero and H_2 has no independent dynamics. For energies much lower than the compositeness scale Λ, instead, H_2 appears as an independent dynamical degree of freedom. Rescaling the field $H_2 \rightarrow H_2(1 - A_0\theta^2)/\sqrt{Z_{H_2}}$, so that it has a canonically normalized kinetic term, the low energy model is given by

$$\begin{aligned}
\Gamma_Z &= \Gamma_{YM} + \int dV \left[\overline{Q} e^{2V_Q} Q + T^C e^{-2V_T} \overline{T}^C + B^C e^{-2V_B} \overline{B}^C\right](1 - \Delta^2\theta^2\overline{\theta}^2) \\
&+ \int dV \overline{H}_1 e^{2V_{H_1}} H_1 (1 - M_H^2 \theta^2\overline{\theta}^2) - \int dS \epsilon_{ij}\left(m H_1^i H_2^j (1 + \delta\theta^2) - h_t H_2^j Q^i T^C\right) \\
&- \int d\overline{S} \epsilon_{ij}\left(m \overline{H}_1^i \overline{H}_2^j (1 + \delta\overline{\theta}^2) - h_t \overline{T}^C \overline{Q}^i \overline{H}_2^j\right) + \int dV \overline{H}_2 e^{2V_{H_2}} H_2 (1 + 2\Delta^2\theta^2\overline{\theta}^2).
\end{aligned} \tag{85}$$

where we have defined the renormalized mass, $m = m_0/\sqrt{Z_{H_2}}$, and Yukawa coupling, $h_t = g_{T_0}/\sqrt{Z_{H_2}}$. Observe that, since m_0 and g_{T_0} have finite values, these renormalized couplings diverge at the scale Λ. Once H_2 is canonically normalized, the effective supersymmetry breaking terms proportional to the bilinear and trilinear terms of the superpotential are $B = \delta$ and $A = 0$, respectively. The negative value of the induced mass parameter for the scalar Higgs H_2 may generate the electroweak symmetry breakdown in the low energy effective theory, even for the case B=0 [28]. However, as mentioned above, a nonvanishing B is necessary in order to induce a nontrivial vacuum expectation value for the scalar Higgs H_1, and therefore the possibility of giving masses to the bottom quarks and leptons of the theory.

The corrections induced by the inclusion of the gauge couplings may be obtained by going to the RG method as in Section 3 [9],[17]. The effects of all the interactions can be obtained by analyzing the modifications to the renormalization group trajectories consistent with the compositeness condition, $Z_{H_2}(\mu = \Lambda) = 0$, which are induced by their inclusion in the low energy theory. It is important to remark, that although the cancellation of the supersymmetry breaking term $A(\mu)$ at all scales is only a property of the bubble sum approximation, the relation $A(\mu)|_{\mu\to\Lambda} = 0$ is a prediction of the model.

The top quark mass value is given by $m_t = h_t(m_t)v_2$, where v_i is the vacuum expectation value of the scalar Higgs H_i. The low energy value of the top quark Yukawa coupling can be obtained by using the renormalization group flows in which h_t becomes large at the compositeness scale Λ. The relevant renormalization group equations in the minimal supersymmetric model are given by [31]:

$$\begin{aligned}
16\pi^2\frac{\partial}{\partial\ln\mu}\alpha_3 &= -6\pi\alpha_3^2 \qquad 16\pi^2\frac{\partial}{\partial\ln\mu}\alpha_2 = 2\pi\alpha_2^2 \qquad 16\pi^2\frac{\partial}{\partial\ln\mu}d\alpha_1 = 22\pi\alpha_1^2\\
16\pi^2\frac{\partial}{\partial\ln\mu}Y_t &= -8\pi^2 Y_t\left(\frac{16}{3}\widetilde{\alpha}_3 + 3\widetilde{\alpha}_2 + \frac{13}{9}\widetilde{\alpha}_1 - 6Y_t - Y_b\right)\\
16\pi^2\frac{\partial}{\partial\ln\mu}Y_b &= -8\pi^2 Y_b\left(\frac{16}{3}\widetilde{\alpha}_3 + 3\widetilde{\alpha}_2 + \frac{7}{9}\widetilde{\alpha}_1 - 6Y_b - Y_t\right)
\end{aligned} \tag{86}$$

where $\alpha_i = g_i^2/4\pi$, $\widetilde{\alpha_i} = \alpha_i/4\pi$, $Y_b = (h_b/4\pi)^2$, $Y_t = (h_t/4\pi)^2$. The solution to these equations provides the renormalization group flow for energy scales $\Delta_S \leq \mu \leq \Lambda$. In general, if supersymmetry is broken at an energy scale Δ_S larger than the electroweak scale, the low energy effective theory is equivalent to the standard model with one or two light Higgs doublets, depending on the value of the mass parameters appearing in the scalar potential. Hence, at scales below Δ_S, the proper renormalization group flow is described by the solutions to the standard model renormalization group equations, which are given by [13]

$$\begin{aligned}
16\pi^2\frac{\partial}{\partial\ln\mu}\alpha_3 &= -14\pi\alpha_3^2 \quad 16\pi^2\frac{\partial}{\partial\ln\mu}\alpha_2 = -2\pi\beta_2\alpha_2^2 \quad 16\pi^2\frac{\partial}{\partial\ln\mu}\alpha_1 = 2\pi\beta_1\alpha_1^2\\
16\pi^2\frac{\partial}{\partial\ln\mu}Y_t &= -8\pi^2 Y_t\left(\frac{24}{3}\widetilde{\alpha}_3 + \frac{9}{4}\widetilde{\alpha}_2 + \frac{17}{12}\widetilde{\alpha}_1 - \frac{9}{2}Y_t - \frac{\alpha_b}{2}Y_b\right)\\
16\pi^2\frac{\partial}{\partial\ln\mu}Y_b &= -8\pi^2 Y_b\left(\frac{24}{3}\widetilde{\alpha}_3 + \frac{9}{4}\widetilde{\alpha}_2 + \frac{5}{12}\widetilde{\alpha}_1 - \frac{9}{2}Y_b - \frac{\alpha_t}{2}Y_t\right)
\end{aligned} \tag{87}$$

where $\beta_2 = 3(19/6)$, $\beta_1 = 7(41/6)$, $\alpha_b = 1(3)$, $\alpha_t = 1(3)$ if there are two (one) light scalar Higgs doublets. In the case in which only one light scalar Higgs doublet

ϕ remains at low energies, it will be given by a combination of the original Higgs doublets H_1 and H_2:

$$\phi = H_1 \cos(\theta_M) + i\tau_2 H_2^* \sin(\theta_M) \tag{88}$$

where θ_M is the mixing angle. As we will show in the next section, when there is only one light Higgs doublet below Δ_S the mixing angle $\tan(\theta_M) = R$, where $R = v_2/v_1$ is the ratio of the Higgs vacuum expectation values. Hence $< \phi >^T = (v, 0)$, where $v = \sqrt{v_1^2 + v_2^2}$. The top and bottom Yukawa couplings appearing in the renormalization group equation (87) are the effective couplings of the doublet ϕ with the top and bottom quarks. These are related to the supersymmetric Yukawa couplings at the scale Δ_S by $h_b^{eff} = h_b \cos(\theta_M)$ and $h_t^{eff} = h_t \sin(\theta_M)$.

For a compositeness scale $\Lambda \gg M_Z$ and a supersymmetry breaking scale which is of the order of the weak scale, an estimate of the top quark mass may be obtained by using the supersymmetric infrared quasi-fixed point, $h_t(M_Z) \simeq \sqrt{\frac{8}{9}} g_3(M_Z)$. This yields a top quark mass approximately equal to $m_t \simeq (196\ GeV) R/\sqrt{1+R^2}$. Consequently, if the ratio $R \simeq 1$, the top quark mass can be significantly lower than the values obtained in the model of Ref.[17].

A more accurate estimate of the top quark mass can be obtained by numerical integration of the renormalization group equations, Eqs.(86)-(87). In Fig. 8, we show the results of ref.[18] obtained for the top quark mass as a function of the ratio R for three different values of the compositeness scale Λ and a supersymmetry breaking scale $\Delta_S = 1\ TeV$. In the numerical work, the compositeness conditions are imposed on the top quark Yukawa coupling $Y_t(\Lambda)^{-1} = 0$. The boundary conditions for the gauge couplings are chosen to be $\alpha_3(M_Z) = 0.115$, $\alpha_2(M_Z) = 0.0336$ and $\alpha_1(M_Z) = 0.0102$ [18]. The low energy bottom Yukawa coupling was fixed by requiring the bottom mass to be consistent with its experimental value $m_b \simeq 5\ GeV$. The values of the gauge and Yukawa couplings at a given energy scale μ are obtained by integrating the renormalization group equations, asking for continuity at the supersymmetry breaking scale Δ_S. The perturbative one loop renormalization group equations may not be reliably used to determine the evolution of the Yukawa couplings at energy scales μ close to the compositeness scale Λ. Again the action of the infrared quasi-fixed point makes the top quark mass predictions very insensitive to the precise high value of the top quark Yukawa coupling at the scale Λ. A slight variation, of less than 1% (2%), of the top quark mass value is obtained by setting $Y_t(\Lambda) = 0.1$, for a compositeness scale $\Lambda \geq 10^{16}$ GeV ($\Lambda \geq 10^{10}$ GeV).

As is apparent from Fig.(8), the minimal value of the top quark mass is obtained for the lowest value of R. In general, the top quark mass values are insensitive to whether only one or two light Higgs doublets appear in the spectrum. However, for low values of R, the top quark mass predictions obtained if there are two light Higgs doublets are slightly lower than those obtained for the one light Higgs doublet case. As we will show in the next section, in the one light Higgs doublet case the ratio R is bounded to be larger than one. In the two light Higgs doublets case, although for characteristic values of the low energy parameters $R \geq 1$, R could be slightly lower than one.

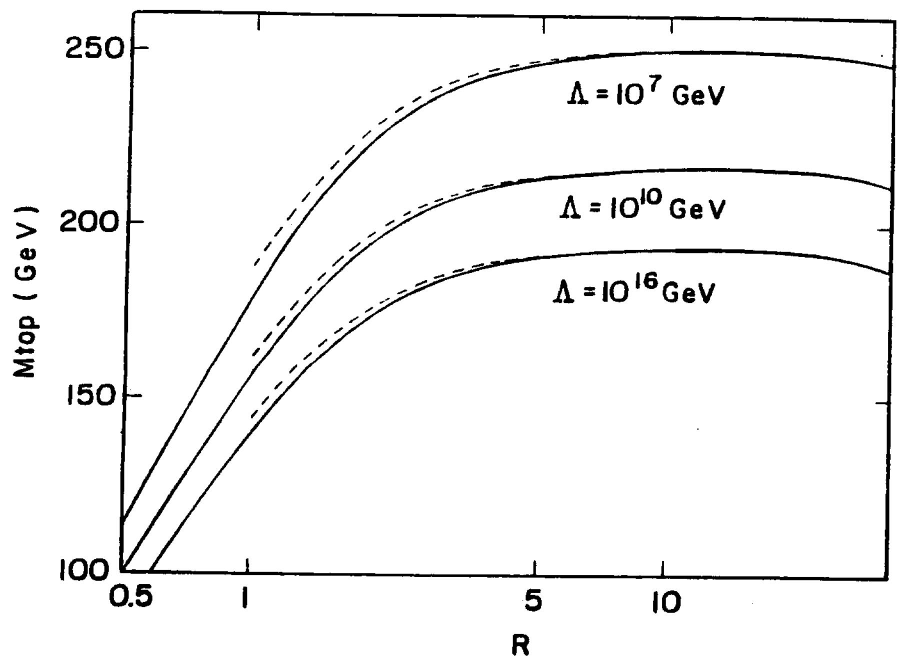

Figure (8): Top quark mass as a function of ratio R for a soft supersymmetry breaking scale $\Delta_S = 1$ TeV, and three values of the compositeness scale Λ for the case of one light Higgs doublet (dashed line) and two light Higgs doublets (solid line).

The top quark mass has the same qualitative behavior for the different values of Λ and Δ_S. Since $v_2(R) = v_2(R=1)R\sqrt{2/(1+R^2)}$, the top quark mass is expected to increase with R, tending to a constant value for large values of R. Such behavior is actually observed for low and intermediate values of R. However, since $m_b = h_b v_1$, the bottom Yukawa coupling depends on R as follows

$$h_b = h_b(R=1)\sqrt{(1+R^2)/2}. \tag{89}$$

Thus, for larger values of R, the bottom Yukawa coupling becomes larger and the infrared quasi-fixed point is reached for lower values of the top Yukawa coupling. In addition, since v_2 varies only slightly with R in the large R regime, the top quark mass decreases with R, a behavior that is clearly seen in Fig.(8). If R becomes too large ($R \geq 36$ for $\Lambda \simeq 10^{16}$ GeV, h_b becomes larger than the top Yukawa coupling. In the computations an upper bound on R is set by requiring the top Yukawa coupling to be larger than h_b.

An important result of these computations is that, if the compositeness scale $\Lambda \simeq 10^{16}\ GeV (10^{10}\ GeV)$, and $\Delta_S \simeq 1\ TeV$, the characteristic top quark mass is 140

GeV $< m_t <$ 195 GeV, (160 GeV $< m_t <$ 220 GeV). Furthermore, the top quark mass results obtained for $\Delta_S = 100$ GeV are very similar to the ones obtained for $\Delta_S = 1$ TeV. For Δ_S as large as 10 TeV, the top quark mass is shifted slightly towards larger values. Therefore, the low energy predictions for the top quark mass obtained in the analysis are stable under variations of Δ_S [18].

We mention that a potential problem for the supersymmetric composite-Higgs models is that a large four Fermi coupling, $\sim 1/\Delta^2$ is imagined to hold on scales up to $\sim \Lambda^2$ and will potentially induce unitarity violations. In leading large–N the unitarizations are performed in the bubble sums, but beyond this approximation the models presented here are really prescriptions for a more general supersymmetric theory, perhaps having a resemblance to a string theory. Whether the underlying theory can generate the couplings required for the condensate models remains an open question. We have not given a review of the phenomenology of the scalar sector and we refer the reader to ref.[18].

7. Fourth Generation and Neutrino Condensate Schemes

Presently we wish to turn to schemes in which electroweak symmetry breaking is driven by a condensate of conventional quarks and leptons, but the scale Λ of new dynamics is not far beyond the electroweak scale. For such a scheme we must invoke a fourth generation. This is apparent already in the analysis of section I, [9], in which one sees that as $\Lambda \to 10$ TeV then $m_t \to 500$ GeV, clearly incompatible with the indirect limits. For a degenerate fourth generation quark doublet, the ρ-parameter limits are not very stringent, and the mass of the fourth generation doublet can be ~ 1 TeV.

In a fourth generation scheme the issue of the non-observation of a fourth neutrino species at LEP and SLC must be faced. If right-handed neutrinos exist, then the most natural explanation for the smallness of the observed left-handed neutrino masses is the see-saw mechanism [33]: Small left-handed neutrino masses are naturally explained by assuming (1) conventional Dirac mass terms for the neutrinos linking left- and right-handed neutrinos and (2) a large Majorana mass term for the right-handed neutrinos. No known gauge interaction is broken by the presence of the large Majorana mass for the right-handed neutrinos. The sterility of the right-handed neutrinos then ensures that the large mass hierarchy between the left- and right-handed masses can be maintained without fine-tuning. After transforming to mass eigenstates, the induced Majorana mass for the left-handed neutrino is of order m_D^2/M_M, where m_D is the Dirac mass and M_M is the Majorana mass.

Thus, with regard to the non–observation of the fourth generation neutrino species we find an intriguing possibility [34]. We assume the existence of a fourth generation, and that: (1) all neutrinos have Dirac masses of order their charged lepton counterpart and (2) all neutrinos have a large right-handed Majorana mass M *of order the* *electroweak scale.* In this scenario, the see-saw mechanism assures that the (e, μ, τ) neutrinos are light while ν_4 is naturally heavy [34]. The fact that M can be taken close to the electroweak scale has been emphasized by Glashow in the context of three generations [35]. Thus, the LEP–SLC limits do *not* imply that there are only three generations of quarks and leptons, even if "neutrino democracy" is invoked. These assumptions also imply that the light neutrinos have masses not far from their current experimental upper limits. In the simplest version which we present here there will be a massive Majorana–Higgs boson and a massless "majoron" associated with the spontaneously broken global right–handed neutrino number [36].

7.2 BCS Theory of the See–Saw Mechanism and the Majoron

We first illustrate the dynamical generation of a Majorana mass for right-handed neutrinos in an NJL model, which contains N generations of right-handed neutrinos ν_{Rj}, where j is the generation index. The Lagrangian is:

$$\mathcal{L} = \bar{\nu}_{Rj} i \partial\!\!\!/ \nu_{Rj} + G_0 (\bar{\nu}^C_{Rj} \nu_{Rj})(\bar{\nu}_{Rk} \nu^C_{Rk}) \ . \tag{90}$$

where repeated indices are summed from 1 to N. Here ψ^c denotes charge conjugation. On scales above Λ the four-fermion interaction softens and is to be viewed to be generated by some new interactions, such as a new gauge interaction. The theory has a global $SO(N)_R \times U(1)$ flavor symmetry. This theory can be solved exactly in the large-N limit, and when G_0 exceeds a certain critical value, there is a vacuum condensate:

$$\langle \bar{\nu}^C_{Rj} \nu_{Rj} + \text{h.c.} \rangle \neq 0 \ , \tag{91}$$

which breaks the $U(1)$, while preserving the $SO(N)$ symmetry, and gives all of the neutrinos a Majorana mass. In addition to giving rise to a Majorana mass, the fact that the $U(1)$ flavor symmetry is spontaneously broken implies that there is a massless Nambu-Goldstone mode (the "majoron") in the spectrum [36]. Also, there is a massive collective mode analogous to the "σ mode" in the NJL model which we will refer to as the Majorana-Higgs boson. In the large-N limit it has a mass exactly twice the neutrino Majorana mass, but there are significant corrections to this result at small N or in the presence of additional interactions.

The solution to the theory defined in eq. (90) may be discussed in an effective Lagrangian framework using the block-spin renormalization group. The renormalized Lagrangian at scales $\mu \ll \Lambda$ can be written:

$$\begin{aligned} \mathcal{L}_\mu \ &= \ \partial^\mu \Phi^\dagger \partial_\mu \Phi - M^2 \Phi^\dagger \Phi - \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 \\ &\quad + \bar{\nu}_{Rj} i \partial\!\!\!/ \nu_{Rj} + \kappa \left(\Phi \bar{\nu}^C_{Rj} \nu_{Rj} + \text{h.c.} \right) + \cdots \ , \end{aligned} \tag{92}$$

where we have introduced the composite (rescaled, auxilliary) field Φ. We can derive the results of compositeness of Φ from the usual one-loop differential RG equations satisfied by the physical couplings. The results are:

$$16\pi^2 \frac{\partial}{\partial \ln \mu}\kappa = (2N+4)\,\kappa^3 , \tag{93}$$

$$16\pi^2 \frac{\partial}{\partial \ln \mu}\lambda = 8N\kappa^2\lambda - 32N\kappa^4 + 8\lambda^2 . \tag{94}$$

The compositeness conditions are just those implied by the bare Lagrangian as in section 3.1:

$$\kappa(\mu) \to \infty|_{\mu\to\Lambda} , \tag{95}$$

$$\lambda(\mu) \to \infty|_{\mu\to\Lambda} . \tag{96}$$

These may be taken as the boundary conditions on the solution to the RG equations. The resulting renormalized coupling constants, κ and λ take the form:

$$\kappa = \left(\frac{N}{8\pi^2}\ln\frac{\Lambda^2}{\mu^2}\right)^{-1/2} , \qquad \lambda = \left(\frac{N}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\right)^{-1} . \tag{97}$$

Again, the fine-tuning of the gap equation is equivalent to demanding an approximate cancellation between the quadratic divergence and the bare Higgs mass at Λ. Thus, when $\mu^2 \to 0$ we demand that $M^2 \to M_\Phi^2$, the desired low energy value of the Φ mass. The interesting physics predictions are then contained in the quantities $\tilde{\lambda}$ and Z_Φ, or equivalently, in λ and κ.

The "predictions" of the model are obtained as follows. We *assume* (as a consequence of our choice of fine-tuning) that the symmetry is spontaneously broken and rewrite for Φ:

$$\Phi = (v_\Phi + \frac{\phi}{\sqrt{2}})e^{i\chi/v_\Phi} , \tag{98}$$

where $\langle\Phi\rangle = v_\Phi$. Here, χ is a massless Nambu–Goldstone mode, the majoron [36], and ϕ is the Majorana–Higgs boson with mass:

$$m_\phi^2 = 2\lambda v_\Phi^2 . \tag{99}$$

Also, we see from the Majorana–Yukawa coupling to the neutrinos,

$$\bar{\nu}_{Rj} i\partial\!\!\!/ \nu_{Rj} + \kappa\left(\Phi\bar{\nu}^C_{Rj}\nu_{Rj} + \text{h.c.}\right) , \tag{100}$$

that we have a Majorana mass for the right-handed neutrinos:

$$m_M = 2\kappa v_\Phi . \tag{101}$$

(Note that m_M is larger by a factor of two than what one naively expects. This comes from deriving the equation of motion for the neutrino field from the Lagrangian, since variations with respect to ν_R and $\bar{\nu}_R$ are not independent.) By using the results for λ and κ from eqs. (2.14) and (2.15) we find that

$$\frac{m_\phi}{m_M} = \sqrt{\frac{\lambda}{2\kappa^2}} = 2 \; . \tag{102}$$

This is the conventional Nambu–Jona-Lasinio result, but m_M is now Majorana, rather than Dirac.

7.3 Incorporation into a Realistic Model

A realistic effective Lagrangian similar to eq. (90) must contain the observed spectrum of quark and lepton masses and mixing angles. The model of ref.[19] contains 4 standard generations of quarks and leptons, together with 4 right-handed neutrinos. At the scale Λ we have a four-fermion effective Lagrangian which may be represented by introducing auxilliary fields H and Φ. The fermions are assumed to have couplings to the auxilliary field H given by:

$$\begin{aligned} \mathcal{L}_{Dirac} \;=\; & g_{jk}^{(-1)}\overline{L}_{Lj}He_{Rk} + g_{jk}^{(0)}\overline{L}_{Lj}\widetilde{H}\nu_{Rk} + g_{jk}^{(2/3)}\overline{Q}_{Lj}Hu_{Rk} + g_{jk}^{(0)}\overline{Q}_{Lj}\widetilde{H}d_{Rk} + \text{h.c.} \\ & - M_{H0}^2 H^\dagger H + \cdots \; , \end{aligned} \tag{103}$$

In addition, we assume that the right-handed neutrinos couple to the auxilliary field Φ:

$$\mathcal{L}_{Majorana} = \kappa_{jk}\left(\Phi\bar{\nu}_{Rj}^C\nu_{Rk} + \text{h.c.}\right) - M_{\Phi 0}^2\Phi^\dagger\Phi + \cdots \; . \tag{104}$$

Here we define $Q_{Li} = (u_L \; d_L)^T$ ($L_{Li} = (\nu_L \; e_L)^T$) to be the ith quark (lepton) electroweak doublet, and $\widetilde{H} = i\sigma_2 H^*$. Note that $\bar{\nu}_j^C\nu_k = \bar{\nu}_k\nu_j^C$ implies $\kappa_{jk} = \kappa_{kj}$. The above ellipses refer to the possible "irrelevant" operators of $d > 4$, such as four-fermion terms that are suppressed by $1/\Lambda^2$ with numerical coefficients of order unity.

Ultimately H and Φ become dynamical fields at low energies and develop vacuum expectation values. Through these VEV's the quarks and leptons acquire Dirac mass terms and the right-handed neutrinos acquire Majorana mass terms. The matrices g_{ij}^α will determine the mass spectrum and the pattern of mixing angles in the hadronic and leptonic weak currents.

We now consider the descent in the full theory to low energies in analogy to our treatment of the BCS–Majorana theory in Section II. The most general induced Lagrangian for both of the the scalar fields is:

$$\begin{aligned} \mathcal{L}_S \;=\; & Z_H(D_\mu H_0^\dagger D^\mu H_0) + Z_\Phi\partial_\mu\Phi_0^\dagger\partial^\mu\Phi_0 - M_{H0}^2 H_0^\dagger H_0 - M_{\Phi 0}^2\Phi_0^\dagger\Phi_0 \\ & - \frac{\widetilde{\lambda}_1}{2}(H_0^\dagger H_0)^2 - \frac{\widetilde{\lambda}_2}{2}(\Phi_0^\dagger\Phi_0)^2 - \widetilde{\lambda}_3 H_0^\dagger H_0\Phi_0^\dagger\Phi_0 \; . \end{aligned} \tag{105}$$

The RG boundary conditions can be derived using the same reasoning as in section 2. As $\mu \to \Lambda$, we demand:

$$Z_\Phi \;\to\; 0 \; ,$$

$$\begin{aligned} Z_H &\rightarrow 0 \, , \\ \widetilde{\lambda}_i &\rightarrow 0 \, , \end{aligned} \tag{106}$$

with all other couplings finite (and nonzero) in this normalization. The masses also evolve as before, but now we assume that the low energy values are such as to trigger the appropriate symmetry breaking as described below. In the physical normalization, $H = Z_H^{1/2} H_0$; and $\Phi = Z_\Phi^{1/2} \Phi_0$ the Lagrangian becomes:

$$\begin{aligned} \mathcal{L}_S &= D_\mu H^\dagger D^\mu H + \partial_\mu \Phi^\dagger \partial^\mu \Phi - M_H^2 H^\dagger H - M_\Phi^2 \Phi^\dagger \Phi \\ &\quad - \frac{\lambda_1}{2} (H^\dagger H)^2 - \frac{\lambda_2}{2} (\Phi^\dagger \Phi)^2 - \lambda_3 H^\dagger H \Phi^\dagger \Phi \, . \end{aligned} \tag{107}$$

with the physical coupling constants defined by:

$$\begin{aligned} \lambda_1 &= \widetilde{\lambda}_1 / Z_H^2, \\ \lambda_2 &= \widetilde{\lambda}_1 / Z_\Phi^2, \\ \lambda_3 &= \widetilde{\lambda}_1 / Z_H Z_\Phi, \\ M_H^2 &= M_{H0}^2 / Z_H, \\ M_\Phi^2 &= M_{\Phi 0}^2 / Z_\Phi. \end{aligned} \tag{108}$$

The boundary conditions can therefore be rewritten as

$$\begin{aligned} \kappa &\rightarrow \infty \, , \\ \lambda_i &\rightarrow \infty \, , \\ d_{44}^\alpha &\rightarrow \infty \, . \end{aligned} \tag{109}$$

The masses M_H^2 and M_Φ^2 are tuned to have low energy values that are negative. This is equivalent to demanding the symmetry breaking solution to the gap equations and thus trigger the formation of the vacuum expectation values of H and Φ. Therefore, we simply parametrize these VEV's at low energies:

$$\left\langle H^0 \right\rangle = v_H = 175 \text{ GeV}; \qquad \langle \Phi \rangle = v_\Phi \equiv \beta v_H. \tag{110}$$

where the parameter β is *a priori* arbitrary.

The Higgs–Yukawa coupling constants will have low energy values:

$$d^{(-1)} = \frac{1}{v_H} \text{diag} \left(m_e, m_\mu, m_\tau, m_{E4} \right) \tag{111}$$

$$d^{(+2/3)} = \frac{1}{v_H} \text{diag} \left(m_u, m_c, m_t, m_{U4} \right) \tag{112}$$

$$d^{(-1/3)} = \frac{1}{v_H} \text{diag} \left(m_d, m_s, m_b, m_{D4} \right) \tag{113}$$

For the neutrinos we make the assumption $d_{ii}^{(0)} \approx d_{ii}^{(-1)}$ for $i = (1,2,3)$, while $d_{44}^{(0)}$ is determined by the RG equations. Here, m_{E4} is the mass of the fourth generation lepton, *etc.* All large coupling constants will be determined in this model in terms of the scale Λ by using the RG equations with the assumption of the compositeness boundary conditions. Taking $d^{(0)} \approx d^{(-1)}$ for the light neutrinos is our special assumption of "neutrino democracy;" we certainly do not predict the three light–mass generation Higgs–Yukawa couplings, but it is reasonable to expect the usual generational hierarchy to apply in the real world for neutrinos. Of course, we allow for the overall scale difference, *i.e.*, $d^{(0)} = \epsilon d^{(-1)}$ with $0.1 \lesssim \epsilon \lesssim 1.0$ as in [34].

The low energy Majorana–Yukawa coupling constants are assumed all to be large and will therefore all be predicted. We will find:

$$\kappa = \mathrm{diag}(\kappa_l, \kappa_l, \kappa_l, \kappa_h) \tag{114}$$

where κ_l refers to the light neutrinos. Hence the light three generations will have approximately degenerate Majorana–Yukawa couplings. $\kappa_h \neq \kappa_l$ arises because of the renormalization effects due to the large Higgs–Yukawa couplings of the fourth generation.

We begin by studying the RG equations that pertain to the fermion Dirac and Majorana masses. In what follows we will shift notation for ease of writing the RG equations. Let us define the matrices:

$$g_{ij}^{(-1)} \equiv E_{ij}; \qquad g_{ij}^{(0)} \equiv N_{ij}; \qquad g_{ij}^{(+2/3)} \equiv U_{ij}; \qquad g_{ij}^{(-1/3)} \equiv D_{ij}; \tag{115}$$

The full one-loop renormalization group equations for the coupling constant matrices as defined above are:

$$16\pi^2 \frac{\partial}{\partial \ln \mu}\kappa = \left[2\,\mathrm{tr}(\kappa^\dagger \kappa) + 4\kappa\kappa^\dagger\right]\kappa + (N^\dagger N)^T \kappa + \kappa N^\dagger N\,, \tag{116}$$

$$\begin{aligned} 16\pi^2 \frac{\partial}{\partial \ln \mu}E &= \Big[\frac{3}{2}EE^\dagger - \frac{3}{2}NN^\dagger + \mathrm{tr}(E^\dagger E) + \mathrm{tr}(N^\dagger N) \\ &\quad + 3\,\mathrm{tr}(UU^\dagger + DD^\dagger) - \frac{15}{4}g_1^2 - \frac{9}{4}g_2^2\Big]E\,, \end{aligned} \tag{117}$$

$$\begin{aligned} 16\pi^2 \frac{\partial}{\partial \ln \mu}N &= \Big[\frac{3}{2}NN^\dagger - \frac{3}{2}EE^\dagger + \mathrm{tr}(E^\dagger E) + \mathrm{tr}(N^\dagger N) \\ &\quad + 3\,\mathrm{tr}(UU^\dagger + DD^\dagger) - \frac{3}{4}g_1^2 - \frac{9}{4}g_2^2 + 2\kappa^\dagger\kappa\Big]N\,, \end{aligned} \tag{118}$$

$$\begin{aligned} 16\pi^2 \frac{\partial}{\partial \ln \mu}U &= \Big[\frac{3}{2}UU^\dagger - \frac{3}{2}DD^\dagger + \mathrm{tr}(EE^\dagger) + \mathrm{tr}(NN^\dagger) \\ &\quad + 3\,\mathrm{tr}(UU^\dagger + DD^\dagger) - \frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\Big]U\,, \end{aligned} \tag{119}$$

$$16\pi^2 \frac{\partial}{\partial \ln \mu} D = \Bigg[\frac{3}{2}DD^\dagger - \frac{3}{2}UU^\dagger + \mathrm{tr}(EE^\dagger) + \mathrm{tr}(NN^\dagger) + 3\,\mathrm{tr}(UU^\dagger + DD^\dagger) - \frac{5}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\Bigg] D\,. \tag{120}$$

Here, g_1, g_2 and g_3 are the $U(1)_Y$, $SU(2)_W$ and $SU(3)$ gauge couplings, respectively. Note that the RG coefficients can be computed in the massless limit. The Feynman rules for ν_R then reduce to the familiar ones for two-component spinors. We have given the equations for arbitrary complex coupling matrices, even though we will assume that the matrices are real and diagonal in what follows.

To simplify the RG equations, we assume that the Yukawa coupling matrices are real and diagonal, and satisfy

$$\begin{aligned} E_{44} \gg E_{jj}\ , \quad N_{44} \gg N_{jj}\ , \quad D_{44} \gg D_{jj}\ , \text{ for } j = 1,2,3\ , \\ U_{44}, U_{33} \gg U_{jj} \text{ for } j = 1,2\ . \end{aligned} \tag{121}$$

This is clearly a good approximation at low energies. The diagonal entries of κ are then split, or equivalently the $SO(4)$ symmetry is broken. It is sufficient to consider only the fourth generation $\kappa_4 \equiv \kappa_{44}$ and the three light generation $\kappa_l \equiv \kappa_{ii}$ independently.

The physical fermion Dirac masses are now determined as:

$$m_{\nu 4} = N_{44}(\mu) v_H \qquad m_E = E_{44}(\mu) v_H \tag{122}$$

$$m_U = U_{44}(\mu) v_H \qquad m_D = D_{44}(\mu) v_H \qquad \mu \sim 100\ GeV, \tag{123}$$

while the Majorana masses are given by:

$$M_{M4} = 2\kappa_h(\mu) v_\Phi = 2\kappa_h(\mu)\beta v_H; \qquad M_{Ml} = 2\kappa_l(\mu)\beta v_H, \tag{124}$$

where again we choose $\mu \sim 100$ GeV as an approximation to the threshold condition that determines the masses, *i.e.*, $m = g(m)v$, but it is sufficient for our purposes. Here, m_E is the mass of the fourth generation charged lepton, and $m_{\nu 4}$ is the Dirac mass of the fourth generation neutrino. M_{M4} is the fourth generation Majorana mass, and M_{Ml} is the Majorana mass of all other neutrinos.

The RG evolution of the light quark and lepton masses is irrelevant insofar as the coupling constants are small. We therefore will use the known values of the Dirac masses for these. For the light neutrinos we will follow [34] and assume that the neutrino Dirac masses are given by $m_\nu = \epsilon m_D$ (*e.g.*, for the muon we assume $m_{\nu\mu} = \epsilon_\mu m_\mu$) where ϵ is an arbitrary parameter.

The physically observable neutrino masses are then:

$$m_{\nu R} = \frac{1}{2}\left[M_M + \sqrt{M_M^2 + 4m_D^2}\right], \qquad m_{\nu L} = \frac{1}{2}\left[M_M - \sqrt{M_M^2 + 4m_D^2}\right]\ , \tag{125}$$

with analogous formulas holding for the first three generations. For the case of the light generations we may use the approximate forms:

$$m_{\nu R} \approx M_l \qquad m_{\nu L} \approx \epsilon^2 m_E^2/M_l. \tag{126}$$

The scalar quartic interactions satisfy analogous RG equations, and we refer the reader to [19]. This analysis is close to that of Suzuki and Luty in considering a two Higgs doublet version of the minimal scheme [37].

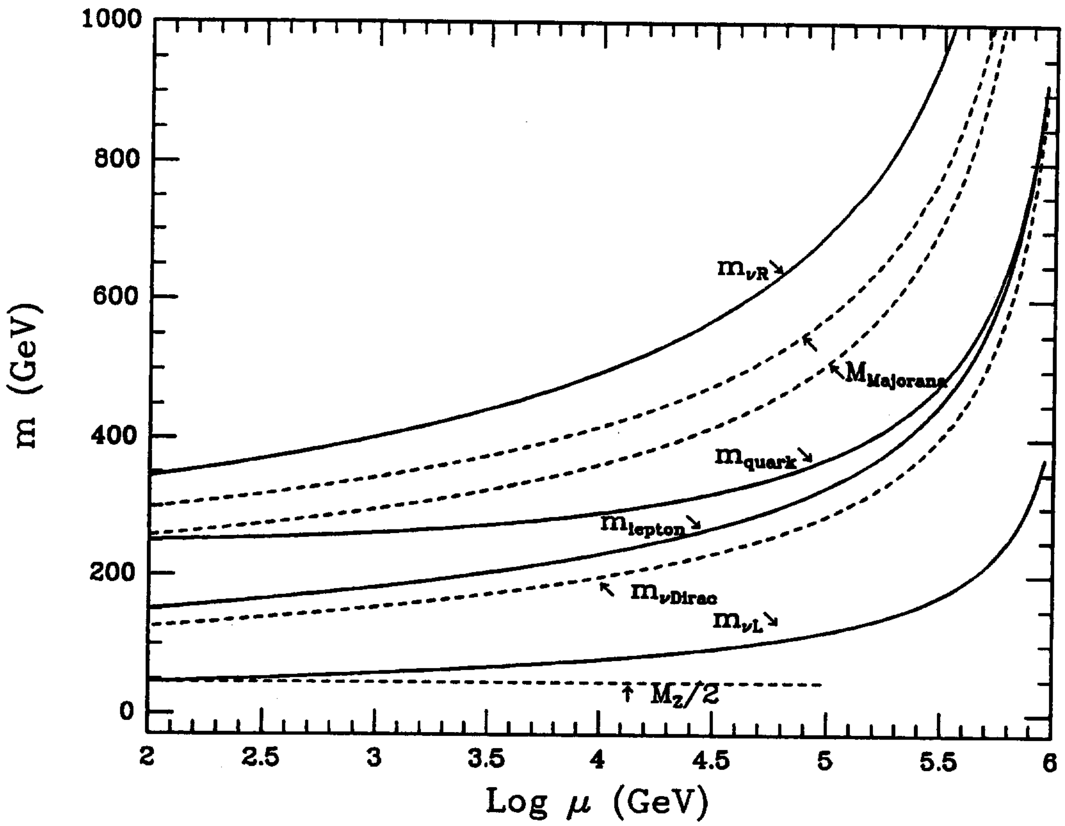

Figure (9): Evolution of Higgs–Yukawa and Majorana–Yukawa coupling constants with scale μ from initial values $g_i = 6$ at $\mu = \Lambda = 10^6$ GeV to $\mu = 100$ GeV. The couplings are translated into masses by multplying by v_H as described in the text. The approach to the infrared fixed points is demonstrated. The larger Majorana masses apply to the light generations.

In Fig.(9) we show the evolution of the Higgs–Yukawa and Majorana–Yukawa coupling constants as a function of scale μ evolving downwards from a compositeness scale of $\Lambda = 10^6$ GeV. We have multiplied all Dirac couplings by v_H, and Majorana couplings by $2v_\Phi = 2v_H$ corresponding to $\beta = 1$. The dashed lines represent the M_h,

M_l and $m_{\nu 4}$ as indicated, while $m_{\nu R}$ and $m_{\nu L}$ are the physically observable values. The purpose of this Figure is to show the attraction from the large initial values down to the low energy fixed points. In practice we used $\kappa_i = d_i = 6$ at $\mu = \Lambda$, but the resulting low energy values are very stable for a wide range of initial conditions. In practice the fourth generation U and D quarks are degenerate to within a few GeV.

In Fig.(10) we show the fourth generation masses as a function of the scale of new physics, Λ, for $\beta = 1$. We have indicated the lower limit $m_{\nu L} \geq M_Z/2$ and we thus see from the Figures *that all schemes are ruled out for sufficiently large* Λ, for example, when $\beta = 1.0$ we require $\Lambda \leq 10^3$ TeV.

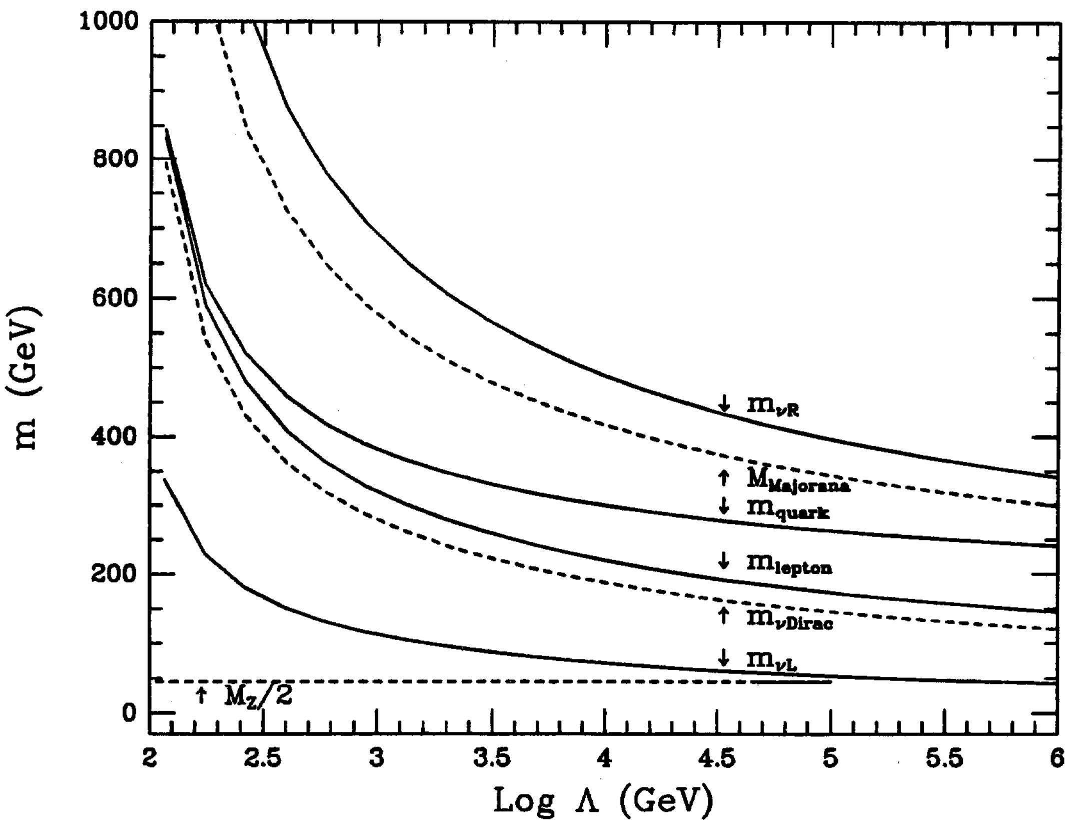

Figure (10): Physical masses (solid lines) as predicted in the model as a function of composite scale Λ, for $\beta = v_\Phi/v_H = 1.0$. The dashed lines indicate the heavy Majorana M_4 and neutrino–Dirac masses $m_{\nu 4}$ separately, before combining to form the physical combinations $m_{\nu R}$ and $m_{\nu L}$.

In order to make definite predictions, we assume throughout that $m_{top} = 130$ GeV. With the latter value of m_t it is unnecessary to consider the evolution of g_{top}, which we then treat as a constant independent of scale. All results are computed at the low energy scale of $\mu = 100$ GeV for simplicity. The largest uncertainties in these results arise from the uncertainty in the non-perturbative running of the Yukawa couplings at high energies. As discussed earlier, this is essentially an uncertainty in the precise high-energy boundary conditions.

In Fig.(11) we give the complete neutrino spectrum as a function of Λ for the case $\epsilon = 1$. Thus, the light neutrino masses as plotted are actually $m_\nu(\beta/\epsilon^2)$. Thus, for $\epsilon_\mu = 0.1$ one must multiply the plotted m_{ν_μ} by 0.01.

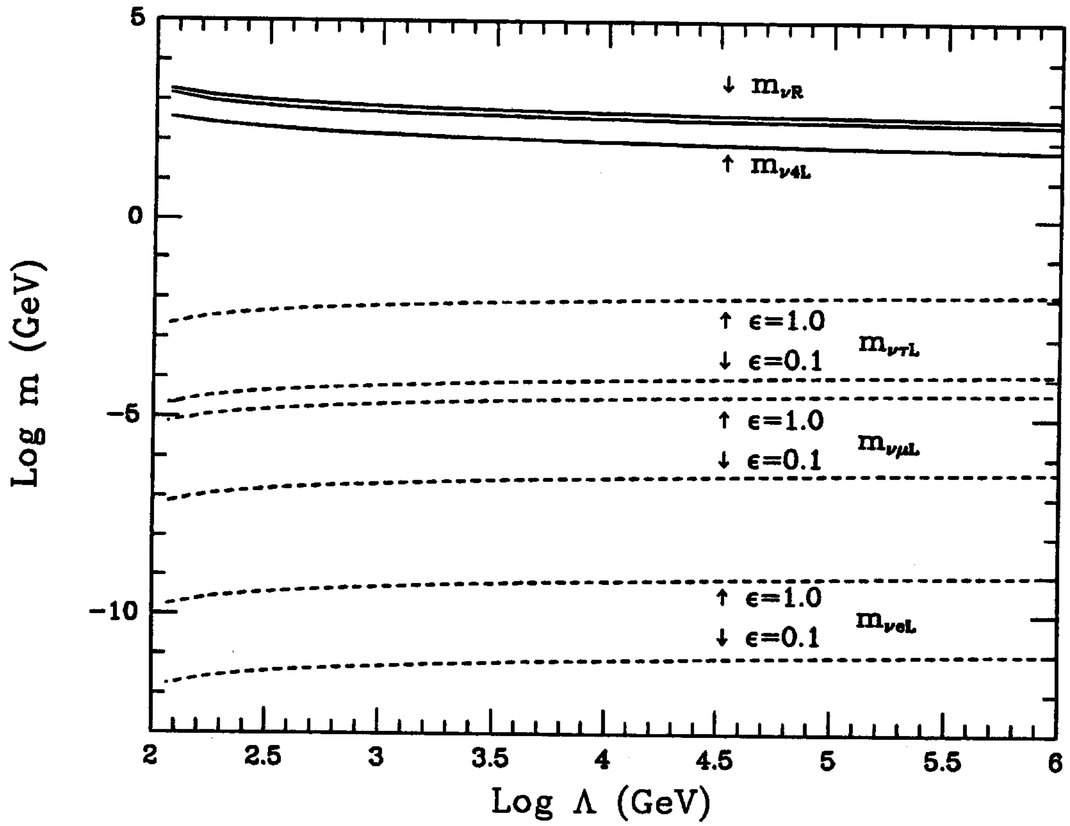

Figure (11): Physical light neutrino masses (solid) as predicted in the model as a function of composite scale Λ for $\beta = v_\Phi/v_H = 1.0$ and we plot for the light masses the range $0.1 \leq \epsilon \leq 1.0$.

The neutrino phenomenology of such a model is expected to be fertile. This requires some further assumptions about mixing angles which we cannot predict in the model. Electroweak phenomenological constraints have also not been considered here in detail. In fact, the "ρ–parameter" constraint should be fairly restrictive, since the top quark mass is already quite sizeable. We have used the central value favored by global parameter analyses of $m_t = 130$ GeV in this analysis. The 90% c.l. upper limit is of order ~ 192 GeV, so at this level we can probably tolerate a charged lepton of order $m_l \lesssim \sqrt{3 \times (192^2 - 130^2)} \sim 240$ GeV, which is a comfortable upper limit in the present model, which predicts $m_{lepton} \sim 182$ GeV for $\Lambda = 10^4$ GeV and $\beta = 1.0$.

8. Conclusions

The main theoretical ideas we have discussed revolve around the notion that conventional quarks or leptons play a fundamental role in the dynamical symmetry breaking of the electroweak interactions. In particular, this provides in the minimal scheme a raison d'etre for the existence of a heavy top quark with a mass of order the weak scale. The predictions of the minimal scheme are completely robust, and very insensitive to the details of the new pairing interactions. The price we pay in such a scheme is the necessity of fine–tuning, which provides the large $\log(\Lambda^2/m_t^2)$, and which ultimately controls the predictions of the model through the infrared fixed points.

The predictions of the minimal scheme yield a top mass of order 230 GeV, which is large compared to current experimental implications through radiative corrections in the electroweak theory. Near future experiments at CDF and D0 will decide the ultimate fate of the minimal top–mode standard model. Nonetheless, this has compelled us to consider the supersymmetric scheme, which allows $m_t > 140$ GeV, and the fourth generation scheme, which does not predict m_t. Both of these schemes have their advantages and flaws. Primarily, they lack the simplicity of the minimal scheme, but they illustrate the fact that the presence of extra degrees of freedom will generally modify the predictions of the minimal scheme, while the general idea of conventional quark and leptons condensates is preserved. There is much more to be done on the theoretical side in this avenue. Electroweak symmetry breaking remains the foremost mystery of particle physics, worthy of continued effort along many lines, including the ideas discussed here.

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